Sequential versus Simultaneous Disclosure

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November 16, 2018

MODEL

- I finite player set.
- Common action set X; X is a finite subset of the real line.
- $x = (x_1, \dots, x_I)$, $x_i \in \mathbb{R}$, let $M(x) = \max\{x_1, \dots, x_I\}$.
- Payoffs $\tilde{u}_i(x) = u_i(M(x))$, where $u_i(\cdot) : X \to \mathbb{R}$ are arbitrary.

Interpretation

Extremely reduced form of a communication game:

1. x_i is "information."
2. Unmodeled Receiver processes information in $x = (x_1, \ldots, x_I)$ and takes an action.
3. $M(x)$ is sufficient statistic for Receiver.
4. Receiver's preferences are increasing in x.
5. Sender's preferences are arbitrary.

Interpretation of Interpretation

1. Bayesian Persuasion (limited set of experiments).
2. Action choice as "facts."

Strong Assumptions

1. Today I'll often assume $u_i(\cdot)$ are one-to-one, but

   the "flavor of the result" holds, . . .

   after dealing with some dirty details.
2. I'll concentrate on dimensional case, but . . .

   arguments generalize to higher dimensions, with weaker bounds.

Two Punchlines

1. Simultaneous Disclosure: After refinement, Senders' favorite equilibrium is salient.
2. There is a way to sequence the Senders so that simultaneous (refined) equilibrium outcome is the same as sequential.

Interpretation

1. Simultaneous disclosure may not be as good as imagined.
2. Sequential disclosure won't be better, but it need not be worse.

Motivation

How does one gather information from many sources.

- Simultaneous (independent consultations; informants don't know what others report).
- Sequential (choice of order is control variable)

Strong Intuition: extra advisers valuable

1. Sometimes non-trivial communication with many, but not with one.
2. Maybe because different advisers know different things (not today).
3. Maybe because of competition.

Full Disclosure Equilibrium in Many Settings

1. Cheap Talk, Senders have identical information. (Two Senders are enough.)
2. Verifiable Information: One sender is enough with strong assumptions on preferences (unraveling). More senders help in general.
3. Bayesian Persuasion: Gentzkow and Kamenica tell you when competition helps (non trivial when Senders have access to different information).

But Perhaps not **that** Valuable

- I study the trivial case. (Senders have the same strategy set.)
- There will “obviously” be a fully revealing equilibrium.
- These results may fail to be robust, convincing because they are against the interest of Senders.
- Result 1: Only Sender’s favorite equilibrium survives IDWDS.

Simultaneous Disclosure is "Superior" to Sequential Disclosure

1. Formally, simultaneous permits full disclosure; sequential generally does not.
2. Intuitively, letting informed agents learn what others report weakens incentive constraints.

Literature

1. Kamenica-Gentzkow
   1.1 One Sender BP.
   1.2 Conditions under which adding Senders helps.
2. Li-Norman
   2.1 General Sequential Bayesian Persuasion
   2.2 Existence, Structure Results, for Fixed Sequence
   2.3 When is Full Information Transmission Possible
   2.4 Comparative Statics: Loosely, adding Senders can hurt, but not if new Sender goes first.
   2.5 (generically) Sequential no more informative than Simultaneous
3. Dekel-Piccone
   Privately, imperfectly informed voters decide on binary decision. Conditions under which sequential disclosure is equivalent to simultaneous.

Roadmap

1. Simultaneous.
2. Sequential.

Pareto Efficiency

Definition

The **smallest strict Pareto disclosure** is

$$\pi^* = \min\{\pi : u_i(\pi) > u_i(x_i) \text{ for all } x_i > \pi \text{ and all } i\}.$$

Definition

The **smallest weak Pareto disclosure** is

$$\tilde{\pi}^* = \min\{\pi : u_i(\pi) \geq u_i(x_i) \text{ for all } x_i > \pi \text{ and all } i\}.$$

1. π^* and $\tilde{\pi}^*$ are well defined.
2. $\pi^* \geq \tilde{\pi}^*$.
3. Equality if $u_i(\cdot)$ is one-to-one for each player.

Simultaneous Disclosure

Agent i selects x_i. Payoffs $u_i(M(x))$

Simple Observations about NE

1. If $x = (x_1, \ldots, x_I)$ satisfies $x_i \leq \pi$ and at least two $x_j = \pi^*$ is a Nash Equilibrium for $\pi = \pi^*$ and $\tilde{\pi}^*$.
2. Full disclosure (maximum x_i) is always NE.
3. Pure-strategy NE are Pareto Ranked: If x^* and x^{**} are both Nash Equilibria and $M(x^*) \leq M(x^{**})$, then $\tilde{u}_i(x^*) \geq \tilde{u}_i(x^{**})$ for all i. This leads us to consider a more restrictive solution concept.

Full disclosure great for the (unmodeled) Receiver, but is the worst NE for Senders.

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Look for refinements.

Iterative Deletion of Weakly Dominated Strategies

Definition

Given subsets $X'_i \subset X_i$, with $X' = \Pi_{i \in I} X'_i$, Player i's strategy $x_i \in X'_i$ is weakly dominated relative to X' if there exists $z_i \in X'_i$ such that $\tilde{u}_i(x_i, x_{-i}) \leq \tilde{u}_i(z_i, x_{-i})$ for all $x_{-i} \in X'_{-i}$, with strict inequality for at least one $x_{-i} \in X'_{-i}$.

Definition

The set $S = S_1 \times \cdots \times S_I \subset X$ survives iterated deletion of weakly dominated strategies (IDWDS) if for $k = 0, 1, 2 \ldots$ there are sets $S^k = S^k_1 \times \cdots \times S^k_I$, such that $S^0 = X$, $S^k \subset S^{k-1}$ for $k > 0$; S^k_i is obtained by (possibly) removing strategies in S^{k-1}_i that are weakly dominated relative to S^{k-1}; $S^k = S^{k-1}$ if and only if for each i no strategy in S^k_i is weakly dominated relative to S^{k-1}; and each S_i can be written in the form $\cap_{k=1}^{\infty} S^k_i$.

Comments

1. Process stops after finitely many steps (finite game).
2. Order generally matters (but not in generic cases).

First Result

Proposition

If x is a strategy profile that survives IDWDS, then $M(x) \in [\tilde{\pi}^*, \pi^*]$. If x is a Nash equilibrium strategy profile that survives IDWDS, then $\tilde{u}_i(x) \geq u_i(\pi^*)$ for all i.

Corollary

If $\pi^* = \tilde{\pi}^*$, then for all x that survives IDWDS, $M(x) = \pi^*$.

Comments

1. Bounds on payoffs that survive refinement (typically strict reduction).
2. Corollary follows directly from Proposition.
3. Corollary says “generically” IDWDS selects Senders’ favorite equilibrium.
4. Non-generic:
   4.1 examples show that multiple outcomes can survive.
   4.2 order of deletion matters
   4.3 no guarantee that π or π^* survives (one or the other will).

Idea of Proof – low disclosures stay

Lemma

There exists a strategy profile $x \in S$ such that $\max\{x_1, \ldots, x_I\} \leq \pi^*$.

If the other Senders aren't disclosing much, you have no reason to disclose a lot.

Idea– very low disclosures leave

Lemma

There exists no strategy profile $x \in S$ such that $M(x) < \tilde{\pi}^*$.

Disclosing $\tilde{\pi}^*$ (or higher) eventually dominates for someone.

Idea – High strategies go

Lemma

No strategy $z_i > \pi^*$ survives IDWDS.

$$z_i = \min\{\arg \min_{x_i \geq \pi^*, x_i \in S_i^k} u_i(x_i)\}$$

is weakly dominated.

Idea – High strategies go

Lemma

No strategy $z_i > \pi^*$ survives IDWDS.

$$z_i = \min\{\arg \min_{x_i \geq \pi^*, x_i \in S_i^k} u_i(x_i)\}$$

is weakly dominated.

honest.
Roughly, z_i is the worst disclosure that hasn't been deleted yet.

Example: Second Result Needs Assumptions

1. Five information structures, 1, 2, 3, 4, 5; higher numbers representing more information.
2. Two Senders.
3. Sender 1 has strict preferences: $2 \succ 4 \succ 1 \succ 5 \succ 3$.
4. Sender 2 has strict preferences: $1 \succ 3 \succ 2 \succ 5 \succ 4$.
5. Unique outcome that survives IDWDS in the simultaneous game is full disclosure: $\pi^* = \tilde{\pi}^* = 5$.

More

Four possible disclosure sequences without returning to a Sender: consulting exactly one Sender, or consulting both in either order. Without commitment, the possible disclosures are:

Sequence	Disclosure
Sender 1	2
Sender 2	1
Sender 1, then 2	1
Sender 2, then 1	2

More claims

Returning to a sender will not lead to more disclosure. Without commitment, sequential disclosure need not lead to disclosure π^*.

Commitment, no return:

- Start with S_1 and sometimes asks S_2, then more disclosure is possible with commitment.
- If the R stops after 1, 2, or 3, S_1 will disclose 2, which will be the final disclosure. If the R stops after 4 or 5, the disclosure will be 4.
- If the R consults S_1 first, then he would do best by committing not to consult S_2 if S_1 discloses at least 4.
- If R consults S_2 first, the disclosure generated will be 1 if the R stops after disclosure 1; 2 if the Receiver stops after 2, 4 or 5, and 3 if the Receiver stops at 3. R can obtain disclosure 3 by consulting first S_2, then S_1 (with commitment).

Conclude

- The best the R can do with commitment but without returning to Senders is disclosure 4.
- Hence commitment increases the disclosure, but does not generate full disclosure.
- But: there is a sequential disclosure protocol that generates disclosure $\pi^* = \tilde{\pi}^*$.
- The protocol involves asking S_1, then S_2, and then going back to S_1, with the commitment to stop if S_2 discloses 3.
- Why? If you start with S_2 and promise to stop after a disclosure of 3 (or more), then the disclosure will be either 3 or 5 (depending on the starting point). This leaves S_1 no choice but to disclose everything.

Preliminaries

1. Histories: $H_0 = \emptyset$, $H_t = X^t$. $H = \cup_{t=0}^{\infty} H_t$.
2. . If $h_t = (h_t^1, \ldots, h_t^t) \in H_t$ and $h_{t'} = (h_{t'}^1, \ldots, h_{t'}^{t'}) \in H_{t'}$ then

$$h_t h_{t'} \in H_{t+t'} : h_t h_{t'} = (h^1, \ldots h^t, h^{t+1}, \ldots, h^{t+t'})$$

where

$$h^m = \begin{cases} h_t^m & \text{if } 1 \leq m \leq t \\ h_{t'}^{m-t} & \text{if } t < m \leq t + t'. \end{cases}$$

Definition

A finite sequential disclosure protocol is a mapping $P : H \to \{0, 1, \ldots, N\}$ such that for all $h, h_t \in H$,

$$P(h_t) = 0 \implies P(h_t h) = 0 \quad (1)$$

and there exists T such that $P(h_T) = 0$ for all $h_T \in H_T$.

Protocol to Game

1. Sender i's strategy specifies a disclosure as a function of h_t for each h_t such that $P(h_t) = i$.
2. $s = (s_1, \ldots, s_n)$ determines disclosures, $d_t^*(s)$ and histories $h_t^*(s)$ for $t = 1, \ldots, T$ where
   - $h_1^*(s) = d_1^*(s) = s_{P(\emptyset)}(\emptyset)$,
   - $d_2^*(s) = s_{P(h_1^*(s))}(h_1^*(s)), h_2^*(s) = h_1^*(s)d_2^*(s)$ and,
   - $d_k^*(s) = s_{P(h_{k-1}^*(s))}(h_{k-1}^*(s)), h_k^*(s) = h_{k-1}^*(s)d_k^*(s)$.
3. π is generated by a sequential disclosure protocol if the induced game has a PBE in which π is disclosed.

Equivalence Result

Proposition

There exists a sequential disclosure protocol that uniquely generates the disclosure $\tilde{\pi}^*$.

Proposition

For all $x > \pi^*$, there exists no sequential disclosure protocol that generates the disclosure x.

Corollary

If $\tilde{\pi}^* = \pi^*$, then there exists a sequential disclosure protocol that uniquely generates the disclosure π^*.

Proof Idea

Induction argument: Given disclosure $x < \tilde{\pi}^*$ some Sender wants to disclose more.

Sequential can as good as simultaneous

Proposition

There exists a sequential disclosure protocol that generates the disclosure π^**.*

Idea: Augment protocol that gets $\tilde{\pi}^*$

Comparing Senders

1. "Obviously" adding a Sender (and using best protocol) doesn't hurt Receiver.
2. Straightforward to describe when one Sender is "more useful" than another (so that the second Sender need not be consulted).
3. Doing this requires formalizing a notion of "closeness of preferences."

How to Organize Information Collection?

(Keri's thesis should include a sole-authored chapter.)

1. Paper gives little guidance.
2. Costly consultation suggests sequential procedures.
3. Why do we sometimes have department meetings (open, non sequential) and sometimes seek second opinions?
4. No motivation in this model to permit Senders to share information, but what if:
   4.1 it is costly to talk (put Senders together so they don't repeat themselves)?
   4.2 Senders have different (complementary) information?
   4.3 Need to motivate information acquisition?

Summary

Two results:

1. Receiver can't rely on simultaneous disclosure.
2. If you are careful, sequential disclosure is no worse (but no better).

Example 1

Example

Consider the following game:

	None	All
None	$1, 1$	$1, 0$
All	$1, 0$	$1, 0$

In this example, $\pi^* = \mathit{All}$ and $\tilde{\pi}^* = \mathit{None}$. The Column Player likes to conceal and the Row Player does not care. If you discard Column's "All" strategy first, you are left with $(\mathit{None}, \mathit{None})$ and $(\mathit{All}, \mathit{None})$. Consequently, both weak and strict Pareto disclosures are consistent with equilibrium (surviving IDWDS).

Example 2

Example

Consider the following game:

	None	Some	All
None	$2, 0$	$1, 2$	$1, 1$
Some	$1, 2$	$1, 2$	$1, 1$
All	$1, 1$	$1, 1$	$1, 1$

In this example, $\pi^* = \mathit{All}$ and $\tilde{\pi}^* = \mathit{Some}$. If you delete the bottom two Row strategies initially you end up with $(\mathit{None}, \mathit{Some})$; if you delete Column's None and All, you end up with $(\mathit{any}, \mathit{Some})$ (so either Some or All is disclosed). You cannot delete $(\mathit{None}, \mathit{Some})$. So the set of disclosures that survive IDWDS always includes $\tilde{\pi}^*$, but may or may not include π^*.

Example 3

Example

Consider the following game:

	None	Some	All
None	$1, 1$	$-1, 0$	$1, 0$
Some	$-1, 0$	$-1, 0$	$1, 0$
All	$1, 0$	$1, 0$	$1, 0$

In this example, $\pi^* = All$ and $\tilde{\pi}^* = None$. if you delete Column's Some and All, then you are left with $(None, None)$ and $(All, None)$. If you delete Row's None and Some, then you are left with (All, Any). You cannot delete Row's All strategy. Here the set of disclosures that survive IDWDS always includes π^*, but may or may not include $\tilde{\pi}^*$.

Example 4

Example

Consider the following game:

	None	Some	All
None	$0, 0$	$0, -1$	$-1, -1$
Some	$0, -1$	$0, -1$	$-1, -1$
All	$-1, -1$	$-1, -1$	$-1, -1$

In this example, $\pi^* = \mathit{All}$ and $\tilde{\pi}^* = \mathit{None}$. $(\mathit{Some}, \mathit{None})$ and $(\mathit{None}, \mathit{None})$ are equilibria that survive IDWDS, but it is weakly dominated to disclose All. Consequently $\tilde{\pi}^*$ is an equilibrium disclosure; π^* is not an equilibrium disclosure; and there is an equilibrium disclosure strictly between $\tilde{pi}^*$ and π^*.

Example 5

Example

Consider the following game:

	None	Some	All
None	2, 1	0, 1	2, 0
Some	0, 1	0, 1	2, 0
All	2, 0	2, 0	2, 0

In this example, $\pi^* = \mathit{All}$ and $\tilde{\pi}^* = \mathit{None}$. The only equilibrium disclosure is π^*.

Example 6

Example

Consider the following game:

	None	Some	All
None	$1, 0$	$-1, 0$	$0, 0$
Some	$-1, 0$	$-1, 0$	$0, 0$
All	$0, 0$	$0, 0$	$0, 0$

In this example, $\pi^* = \mathit{All}$ and $\tilde{\pi}^* = \mathit{None}$. Row's "Some" strategy is weakly dominated, but all other strategies survive. Consequently there exists an outcome that survives IDWDS in which Row's payoff is less that π^*. This outcome $((\mathit{None}, \mathit{Some}))$ is not an equilibrium.

Summary

The examples show that when there are ties the order of deleting strategies matter. We cannot guarantee that disclosure π^* or disclosure $\tilde{\pi}^*$ will survive IDWDS nor can we guarantee that all payoffs that survive IDWDS are greater than or equal to $u_i(\pi^*)$. We will show that for each strategy x that survives IDWDS, $M(x) \in [\tilde{\pi}^*, \pi^*]$, equilibrium utilities are at least $u_i(\pi^*)$, and that when $\pi^* = \tilde{\pi}^*$ any strategy that survives IDWDS leads to disclosure π^*.