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**Sources of Productivity Growth:  
Technology, Terms of Trade,  
and Preference Shifts**

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# Sources of Productivity Growth: Technology, Terms of Trade, and Preference Shifts\*

*Thijs ten Raa<sup>†</sup>, Pierre Mohnen<sup>‡</sup>*

## Résumé / Abstract

D'habitude, on mesure la croissance de la productivité par le résidu de Solow. Pour ce faire, on a besoin de prix et de parts de facteurs. Puisque ces prix sont supposés être égaux aux productivités marginales, la mesure habituelle prend pour acquis ce qu'elle est censée mesurer. Dans cet article, nous déterminons la croissance de la productivité totale des facteurs sans avoir recours à des données sur les prix des facteurs. Les productivités factorielles sont définies comme des multiplicateurs de Lagrange d'un programme qui maximise le niveau de la demande finale domestique. La mesure qui découle de la croissance de la productivité totale des facteurs inclut non seulement le résidu de Solow, mais aussi les effets dus aux termes de l'échange et aux changements de préférence. En utilisant les tableaux entrée-sortie canadiens de 1962 à 1991, nous montrons que la source de la croissance de la productivité au Canada est passée du changement technique aux améliorations des termes de l'échange.

*The standard measure of productivity growth is the Solow residual. Its evaluation requires data on factor input shares or prices. Since these prices are presumed to match factor productivities, the standard procedure amounts to accepting at face value what is supposed to be measured. In this paper we determine total factor productivity growth without recourse to data on factor input prices. Factor productivities are defined as Lagrange multipliers to the program that maximizes the level of domestic final demand. The consequent measure of total factor productivity is shown to encompass not only the Solow residual, but also the terms-of-trade and preference-shift effects. Using input-output tables from 1962 to 1991 we show that the source of Canadian productivity growth has shifted from technical change to terms-of-trade effects.*

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# 1 Introduction

This is a methodological paper. The measurement of total factor productivity (TFP)-growth constitutes a conceptual puzzle. It involves the use of wage and rental rates to construct an input aggregate. The growth rate of the latter is compared with the growth rate of output. When output grows faster than input, there is productivity growth. Estimates of productivity growth are used to define the ‘room’ in collective wage bargaining. However, since the underlying TFP measure hinges on observed wage and rental rates, there is some circularity in the reasoning.

The puzzle is resolved for perfectly competitive economies. In such economies factor inputs are rewarded according to their marginal productivities. TFP can be conceived as the sum of these marginal productivities taken over all factor inputs. The consequent growth rate of TFP yields the Solow residual measure of TFP-growth, defined as the growth rate of output minus the value-share weighted sum of the growth rates of the inputs. Solow (1957) and Jorgenson and Griliches (1967) have shown the equivalence of TFP-growth with the shift of the production possibility frontier under perfect competition.

The trouble is, however, that observed economies are not perfectly competitive. They are not even on their production possibility frontiers. If we nonetheless stick to the conventional measures of TFP-growth, employing observed value shares for labor and capital, it is not clear what we get. The residual no longer isolates technical change effects, but also captures variations of the economy about the competitive benchmark, such as changes in market power, returns to scale or the business cycle. One approach followed in the literature is to correct the Solow residual for those departures from perfect competition, estimating mark-ups over marginal cost, scale elasticities and utilization rates, and modifying the formula for the residual (Morisson, 1988, and Hall, 1990). Traditional TFP measures moreover, take prices and resource allocations in the economy as given. In other words, the mainstream approach is one of partial equilibrium.

Rather than taking prices and quantities as they are observed and trying to get a handle on the various departures from perfect competition, we adopt a different approach. We define TFP growth as a shift of the production

possibility frontier of the economy which is itself determined from the fundamentals of the economy. The prices implicit in the TFP calculations are the general equilibrium factor prices supporting the production possibility frontier and not the observed factor prices. The fundamentals are the usual ones: endowments, technology, and preferences. Endowments are represented by a labor force and stocks of capital. Technology is given by the combined inputs and outputs of the sectors of the economy. Preferences are reflected by the pattern of domestic final demand.

The productivities are determined as follows. We maximize the level of domestic consumption subject to material balances and endowment constraints. Now, as is known from the theory of mathematical programming, the Lagrange multipliers associated with the endowment constraints measure the marginal productivities of labor and capital: the consumption increments per units of additional labor or capital. In economics, these Lagrange multipliers are shadow prices that would reign under idealized conditions of perfect competition. We declare these shadow prices to be the factor productivities.

The main theoretical contributions of our paper are two. First, we demonstrate that the Lagrange multipliers foundation of factor productivities reconciles the frontier approach with the growth accounting literature. The reconciliation is mutually beneficial. As mentioned, the growth accounting literature suffers from some circularity in the reasoning as it employs observed wage and rental rates. The frontier approach has the potential of determining these values. Conversely, the frontier approach uses a mechanical output measure, is unable to ascribe TFP to labor or capital, and lacks an interindustry analysis. We insert an economic criterion in its mathematical program and thus enrich the frontier approach with all the useful ingredients of mainstream TFP-analysis.

We even go a step further. As is well known, growth can be decomposed into a movement of the frontier, usually defined by the best-practice economy (the U.S.), and a movement towards the frontier, reflecting catch-up. We do not need a best-practice benchmark, but let the frontier, or potential gross domestic product (GDP), be determined by the optimal allocation of resources. A movement of the frontier reflects a change in the structure of the economy (technology, terms of trade, and preferences) and a movement

towards the frontier reflects an allocative efficiency gain. Only a general equilibrium model can relate TFP-growth to the structure of the economy without recourse to observed factor input shares or prices. Since, by the second welfare theorem, the frontier is supported by competitive prices, as production possibility sets are convex and externalities absent, a competitive model is appropriate to determining the production possibility frontier. In the market place monopoly power and other departures may distort prices and quantities, but this is irrelevant for the measurement of the production possibility shift.

The second theoretical contribution of our paper is that it discloses the terms-of-trade effect in productivity analysis. It is well known that an improvement in the terms of trade is equivalent to technical progress. In this paper we will demonstrate that TFP-growth based on growth rates of the Lagrange multipliers can be decomposed into technical change, preference shift, and terms-of-trade effects. Most of the literature implicitly assumes an aggregated output and, therefore, is unable to detect preference shifts or terms-of-trade effects, identifying TFP-growth with the Solow residual measure of technical change. Diewert and Morrison (1986) capture terms-of-trade effects, but they classify commodities a priori as exports or imports. This is not a tenable assumption over a long period of analysis.<sup>1</sup> We shall overcome these obstacles, by letting trade be free, including its direction. Our general equilibrium model detects preference and terms-of-trade effects as integral parts of TFP-growth, without calculations "on the side."

The paper is organized as follows. Factor productivities and TFP are defined by means of a linear program in the next section. In section 3 we apply our methodology to the Canadian economy in the period from 1962 to 1991. The last section concludes.

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<sup>1</sup>Moreover, they assume a jointness in outputs that precludes specialization and even violates global convexity in production, an assumption needed for their analysis.

## 2 General equilibrium analysis of productivity

We push the economy to its frontier by maximization of the level of domestic final demand, which excludes trade by definition. Exports and imports are endogenous, controlled by the balance of payments. We make no distinction between competitive and non-competitive imports.<sup>2</sup>

Domestic final demand comprises consumption and investment. Investment is merely a means to advance consumption, albeit in the future. We include it in the objective function to account for future consumption. In fact, Weitzman (1976) shows that for competitive economies domestic final demand measures the present discounted value of consumption. In principle, our methodology could accommodate endogenous investment and the determination of the intertemporal production possibility frontier as in Hulton (1979), but we have not pursued this approach.

Although we assume Leontief production and utility functions, there is extensive substitutability of factor inputs as we allow for free trade and factor mobility. (The economy may even feature a Cobb-Douglas macro-economic production function, as demonstrated in ten Raa, 1995.)

Productivity growth is defined as the measure of the shift of the frontier. Instead of comparing observations of the economy in subsequent periods, we compare the projections on the respective frontiers. This definition of productivity growth is in line with Solow (1957). He implicitly assumed that the economy is on its frontier. We do not do so, but will push it to the frontier. The expansion factor, which will be denoted by  $c$  in program (1) below, accounts for the transition from observed GPD to potential GDP. The distinction between shifts of the frontier and movements towards the frontier (elimination of inefficiency) is reminiscent of the work by Noshimizu and Page (1982) and is often encountered in the DEA literature (such as in Perelman, 1995). We analyze shifts of the frontier only.

We normalize the level of domestic final demand using base year price. The

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<sup>2</sup>Non-competitive inputs are indicated by zeros in the make table.

primal program reads

$$\begin{aligned}
 & \max_{s,c,g} (e^\top f + k^\circ + l^\circ)c \text{ subject to} \\
 & (V^\top - U)s \geq fc + Jg =: F \\
 & Ks + kc \leq M \\
 & Ls + lc \leq N \\
 & -\pi g \leq -\pi g^t =: D \\
 & s \geq 0.
 \end{aligned} \tag{1}$$

Here the variables ( $s, c$  and  $g$ ) and parameters (all other) are the following [with dimensions in brackets].

$s$	activity vector [# of sectors]
$c$	level of domestic final demand [scalar]
$g$	vector of net exports [# of tradeable commodities]
$e$	unit vector of all components one
$\top$	transposition symbol
$f$	domestic final demand [# of commodities]
$k^\circ$	base-year rent for non-business capital [scalar]
$l^\circ$	base year bill for non-business labor [scalar]
$V$	make table [# of sectors by # of commodities]
$U$	use table [# of commodities by # of sectors]
$J$	0-1 matrix placing tradeables [# of commodities by of tradeables]
$F$	final demand [# of commodities]
$K$	capital stock matrix [# of capital types by # of sectors]
$k$	non-business capital stock [# of capital types]
$M$	capital endowment $Ke + k$ [# of capital types]
$L$	labor employment row vector [# of sectors]
$l$	non-business labor employment [scalar]
$N$	labor force [scalar]
$\pi$	U.S. relative price row vector [# of tradeables]
$g^t$	vector of net exports observed at time t [# of tradeables]
$D$	observed trade deficit [scalar].

Productivities are not measured by market prices, but are determined by the dual program, which, as is well known, solves for the Lagrange multipliers of the primal program. These measure the marginal products of the objective value with respect to the constraining entities, unlike observed factor rewards

with all their distortions. The dual program reads

$$\begin{aligned}
& \min_{p,r,w,\epsilon \geq 0} rM + wN + \epsilon D \text{ subject to} \\
& p(V^\top - U) \leq rK + wL \\
& pf + rk + wl = e^\top f + k^\circ + l^\circ \\
& pJ = e\pi.
\end{aligned} \tag{2}$$

The variables in the dual program are shadow prices:  $p$  of commodities,  $r$  of capital (# of capital types),  $w$  of labor and  $\epsilon$  of foreign debt (the exchange rate). Since the commodity constraint in the primal program has a zero bound,  $p$  does not show up in the objective function of the dual program.  $p$  is normalized by the second dual constraint, essentially about unity.<sup>3</sup>

We now introduce the concept of productivity growth. Since labor productivity is the Lagrange multiplier or shadow price associated with the labor constraint,  $w$ , labor productivity growth is the growth of  $w$ ,  $\dot{w} = dw/dt$ . Similarly,  $r$  is the vector of marginal productivities for each type of capital stock and  $\epsilon$  the marginal productivity of the trade deficit. Total factor productivity (TFP)-growth is obtained by summing all factor productivity growth figures over endowments,  $\dot{r}M + \dot{w}N + \dot{\epsilon}D$ , and normalizing by the level of productivity,  $rM + wN + \epsilon D$ . Formally,

**Definition.**

$$\text{TFP-growth} = (\dot{r}M + \dot{w}N + \dot{\epsilon}D)/(rM + wN + \epsilon D). \tag{3}$$

**Remark.** Replacement of  $(f, k^\circ, l^\circ)$  by  $(\lambda f, \lambda k^\circ, \lambda l^\circ)$  in the primal program with  $\lambda > 0$  yields solution  $(s, c/\lambda, g)$ . The value of the objective function is not affected. By the main theorem of linear programming the values of (1) and (2) are equal, hence  $rM + wN + \epsilon D$  is not affected either. Hence, the productivities are unaffected, as is, by extension, TFP-growth.

The above straightforward definition of TFP-growth is now related to the commonly used Solow residual. By the main theorem of linear programming substituting the second constraint of (2), we obtain the macro-economic iden-

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<sup>3</sup> $p$  is not a device to convert nominal values to real values, but a price vector that sustains the optimal allocation of resources in the linear program.

tity of national product and income (apart from the net exports on either side):

$$pfc + rkc + wlc = rM + wN + \epsilon D. \quad (4)$$

By total differentiation of (4):

$$\text{TFP-growth} = [(pfc + rkc + wlc)' - r\dot{M} - w\dot{N} - \epsilon\dot{D}]/(pfc + rkc + wlc) \quad (5)$$

To establish the link with the Solow residual, focus on the numerator, using (1),

$$(pF - pJg + rkc + wlc)' - r(Ks + rkc)' - w(Ls + lc)' + \epsilon(\pi g)'. \quad (6)$$

Differentiating products, rearranging terms, and using the dual constraint and the definition of  $F$  presented in the primal program, (1), we obtain

$$\begin{aligned} & p\dot{F}' - r(Ks)' - w(Ls)' \\ & - pJ\dot{g}' + \epsilon(\pi g)' \\ & + \dot{p}(F - Jg)' + (rkc)' - r(kc)' + (wlc)' - w(lc)' \\ & = \\ & p\dot{F}' - r(Ks)' - w(Ls)' \\ & + \epsilon\dot{\pi}g \\ & + \dot{p}fc + \dot{r}kc + \dot{w}lc. \end{aligned} \quad (7)$$

We now have a surprising three-way decomposition of total factor productivity growth.<sup>4</sup> Technical change is represented by only one term, the first one, that is the *Solow residual (SR)*. In remark 4 below it will be shown that it can be expressed as a weighted sum of sectoral Solow residuals, where the weights change over time as final demand composition effects move the relative importance of sectors (Wolff, 1985). The second term,  $\epsilon\dot{\pi}g$ , represents the *terms-of-trade effect*. Since proportional changes  $\pi$  are offset by a change in  $\epsilon$ , only relative international price changes matter. The last term is the *preference shift* effect. To reveal it more closely, recall that  $pf + rk + wl$  may be held constant by appropriate choice of  $\lambda$  in the remark following (3),

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<sup>4</sup>Strickly speaking, there should be a fourth term with the slack changes, because in deriving (6) all constraints are assumed to be binding.

so that the preference shift effect, the last line in (7), may be rewritten as  $-(pf + rk + wl)c$ . This expression is positive if the pattern of domestic final demand,  $(f, k, l)$ , shifts towards commodities with low opportunity costs. Then it becomes easier to satisfy the needs and, therefore, TFP is boosted. This preference shift effect comes on top of the just mentioned Wolff (1985) demand effect.

Only relative price changes drive the terms-of-trade and preference shift effects. In other words, these effects do not show in a pure macro-economic setting with only one commodity and no non-business income.<sup>5</sup> Then our measure of TFP-growth coincides with the Solow residual.<sup>6</sup>

**Examples.** In three examples we will highlight the technical change, terms-of-trade, and preference-shift components of TFP-growth. The first two examples feature no trade, but ascribe all TFP-growth to either the Solow residual or the taste effect. The third example illustrates the terms of trade effect. The examples differ by end situation. The base situation is always an economy with labor inputs  $L = \left(\frac{4}{3}, \frac{2}{3}\right)$  and commodity outputs  $V = I$ . There are no trade, capital, intermediate inputs, or unemployed labor.

In the *first* example, output shifts from commodity 2 to commodity 1, so that  $V$  turns  $\begin{pmatrix} 1 + \delta & 0 \\ 0 & 1 - \delta \end{pmatrix}$ . The primal program reads

$$\begin{aligned} & \max(1 + \delta + 1 - \delta)c \text{ subject to} \\ & \begin{pmatrix} (1 + \delta)s_1 \\ (1 - \delta)s_2 \end{pmatrix} \geq \begin{pmatrix} (1 + \delta)c \\ (1 - \delta)c \end{pmatrix} \\ & \frac{4}{3}s_1 + \frac{2}{3}s_2 \leq 2 \\ & s \geq 0. \end{aligned}$$

The solution is  $s_1 = s_2 = c = 1$  with value 2 for the objective function, both in the base situation ( $\delta = 0$ ) and the end situation. By the macro-economic

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<sup>5</sup>With only one commodity and no non-business income,  $\pi$  and  $p$  by the second dual constraint are unity and hence their derivatives vanish.

<sup>6</sup>A tiny difference remains in the denominators. As we divide by  $pf + rk + wl = pF - pJg + rk + wl = pF - \epsilon\pi g + rk + wl = pF + \epsilon D + rk + wl$ , we correct for the deficit and non-business incomes. This correction is minor.

identity  $w$  was and is 1. Hence TFP-growth as defined in (3) is zero. There is technical change, however, for output has shifted towards the resource intensive commodity, stepping outside the initial production possibility frontier. The numerator of the Solow residual is  $p\dot{F} = \begin{pmatrix} \frac{4}{3} & \frac{2}{3} \\ +\delta & -\delta \end{pmatrix} = \frac{2}{3}\delta$ . This is basically the demand composition effect stressed by Wolff (1985). The new demand is unfavorable. The preference shift effect is  $\dot{p}fc$ . Since  $s$  is positive (by the material balance), the first dual constraint is binding (by complementary slackness), so that the price vector turns  $\left(\frac{4/3}{1+\delta} \quad \frac{2/3}{1-\delta}\right)$  and, therefore, has derivative  $\left(-\frac{4}{3}\delta \quad \frac{2}{3}\delta\right)$  (for  $\delta$  small), so that the preference shift effect is  $\left(-\frac{4}{3}\delta \quad \frac{2}{3}\delta\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  (for  $\delta$  small) or  $-\frac{2}{3}\delta$ .

The *second* example is similar, but now  $V$  turns  $\begin{pmatrix} 1-\delta & 0 \\ 0 & 1+2\delta \end{pmatrix}$ . The solution to the primal program becomes  $(1-\delta+1+2\delta) * 1 = 2+\delta$  and the wage rate becomes  $1+\frac{\delta}{2}$  to satisfy (4). The gain,  $\frac{\delta}{2}$ , has to be multiplied by the number of workers, 2, yielding a TFP-growth of  $\delta$ . It can be ascribed entirely to the preference-shift effect, for the economy shifts along its frontier, foregoing  $\delta$  of the doubly labor intensive commodity, nr. 1, for  $2\delta$  of commodity nr. 2. Hence the Solow residual is zero.

Trade is introduced in the *third* example, where the only change is that world prices  $(1 \ 1)$  turn  $(1+\delta \ 1-\delta)$ . The linear program expands the domestic consumption vector,  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ , by letting the economy specialize in the resource extensive commodity, nr. 2. Output is the same before and after the international price change, but the terms of trade deteriorate, reducing the level of consumption and, therefore, the real wage rate and TFP.

### Remarks.

**1.** The TFP measure used in Mohnen, ten Raa and Bourque (1997) is confined to the Solow residual without the terms-of-trade and preference-shift effects. It was derived from total differentiation of the complementary slackness conditions of the first constraint of (2). There is also a slight normalization difference. In this paper, we normalize with respect to  $rM + wN + \epsilon D = pfc + rkc + wlc$ , whereas Mohnen, ten Raa and Bourque

(1997) normalize with respect to  $pF = pfc + pJg$ .

**2.** Implicit in our model is the assumption of Leontief preferences over domestic final demand. Retail and banking services are components of the domestic final demand vector. In a way, one might argue that households favor reductions of these components. The smaller the margins, the more efficient the economy. This effect is captured by the preference shift effect component of TFP-growth. Factor productivity gains within these service sectors are captured by the Solow residual.

**3.** In discrete time, the expressions involving differentials are approximated using the identity  $x_t y_t - x_{t-1} y_{t-1} = \hat{x} \bar{x}_t \bar{y}_t + \hat{y} \bar{x}_t \bar{y}_t$ , where  $\hat{x}_t = (x_t - x_{t-1})/\bar{x}_t$  and  $\bar{x}_t = (x_t + x_{t-1})/2$ , and similarly for  $\hat{y}_t$  and  $\bar{y}_t$ .

**4.** By Domar's aggregation we can decompose the aggregate Solow residual into sectoral and group-sectoral Solow residuals. Let  $j$  index the sectors,  $i$  the commodities, and  $k$  the sector groups. Denote a relative growth rate by  $\hat{L}_j = \dot{L}_j/L_j$ . Define the Solow residual of group-sector  $k$  as:<sup>7</sup>

$$SR_k = \sum_{j \in k} (\sum_i p_i v_{ji} s_j \hat{v}_{ji} - \sum_i p_i u_{ij} s_j \hat{u}_{ij} - w L_j s_j \hat{L}_j - \sum_i r_i K_{ij} s_i \hat{K}_j) / \sum_{j \in k} \sum_i p_i v_{ji} s_j \quad (8)$$

Notice that if  $k = j$ , we get the Solow residual for sector  $j$ . It can be shown that our aggregate Solow residual ( $SR$ ) expression can be written as:

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<sup>7</sup>The numerator of the Solow residual of sector  $j$  is

$$\sum_i p_i [(v_{ji} - u_{ij}) s_j] - \sum_i r_i (K_{ij} s_j + \sigma_{ij}) - w (L_j s_j).$$

The product rule of differentiation yields the term in  $SR_k$ , plus, strictly speaking,

$$\sum_{j \in k} [\sum_i p_i (v_{ji} - u_{ij}) - \sum_i r_i K_{ij} - w L_j] s_j - \sum_{j \in k} \sum_i r_i \hat{\sigma}_{ij}$$

The first term can be interpreted as a structural change effect, contributing to productivity growth through the activation of profitable sectors or the inactivation of unprofitable vectors. The second term translates into productivity effects due to the reduction of idle resources.

$$SR = \frac{\sum_k \sum_{j \in k} \sum_i p_i v_{ji} s_j}{\sum_i p_i F_i} SR_k. \quad (9)$$

### 3 An application to the Canadian economy

To illustrate our methodology, we examine producing growth in the Canadian economy during the period from 1962 to 1991 at the medium level of disaggregation, which comprises 50 industries and 94 commodities. The linear program was solved for each year from 1962 to 1991 yielding the optimal activity levels and shadow prices for the TFP-expressions.

Table I contains the shadow prices of labor (in 1986 \$/hour), of the three types of capital, and of the trade deficit (the latter four are in 1986\$/1986\$, that is rates of return) from 1962 to 1991. Labor was worth at the margin \$16.13 in 1986 prices in 1962. Its productivity followed an increasing trend until 1982 and then a bumpy road ending at \$46.13 in 1991. The rate of return on buildings followed a downward trend, dropping to zero in 1982, sharply rebounded in 1984, and then dropped again to reach zero from 1988 on. In other words, there were excess buildings in 1982 and in 1988-1991. Equipment was not fully utilized until 1983 and again in 1988, 1990 and 1991. Comparing the evolutions of their shadow prices, labor, buildings and equipment seem to be substitutes. Infrastructure had an increasing rate of return until 1974, much greater than the other two types of capital, and then a declining productivity until the end of our period. On average over the 1962-1991 period, a dollar increase in the trade deficit allowed final demand to buy 64 cents.<sup>8</sup> Its shadow price was pretty stable until 1981 and more volatile and somewhat lower after 1981.

Following the conception proposed in this paper, to consider TFP-growth as the sum of factor productivity growths where the latter are determined by the Lagrange multipliers of the endowment constraints, Table 2 shows TFP-growth by factor input. In the first period, 1962-1974, TFP grows a healthy

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<sup>8</sup>Final demand does not increase by the full dollar because of the need to produce locally non-tradeable commodities for a given commodity composition of final demand.

2.6 percent a year.<sup>9</sup> The second period, 1974-1981, shows the notorious slowdown, in fact a negative TFP-growth of -0.5 percent a year. The last period, 1981-1991, TFP rebounded to 3.8 percent a year. The bulk of TFP-growth is attributed to labor, next to nothing to the trade deficit, and the remainder to capital. In the first period the 2.6% TFP-growth consists of 2.4% labor productivity growth and 0.2% capital productivity growth, according to the first column of Table 2. The latter is distributed very unevenly over the three types of capital, with infrastructure picking up 1.1%, equipment none, and buildings plummeting by -0.9%. The slowdown in the second period is ascribed to both labor (dropping to 0.5% a year) and capital (turning -1.0% a year). As in the first period, infrastructure is decisive, now explaining all of the negative productivity growth in the second period. The successful TFP-growth in the last period is a labor story. Labor productivity growth was a dramatic 5% a year, offsetting a reduction in capital productivity growth of 1% a year. Again, the latter is determined by the productivity of infrastructure.

While Table 2 shows the composition of TFP-growth by factor input, Table 3 decomposes it into the three sources of frontier shift, namely technical change, the terms-of-trade effect and the shift in preferences. The first line of Table 3 is identical to the first line of Table 2. In the first period the bulk of TFP-growth (2.6%) is caused by technical change (the Solow residual at shadow prices is 1.7%). The TFP slowdown in the second period is also ascribed to a downturn in technology. The recovery in the last period, however, is due not only to a Solow residual (at shadow prices) increase of one percent, but above all to an improvement in the terms-of-trade effect from 0.5 to 3.8% annually. It might look strange to have some negative Solow residuals, albeit at shadow prices. How can technology regress? There are at least three serious explanations to it. First, technical progress does not show in the statistics right away. This is the argument raised by David (1990) to explain the productivity paradox. It takes time to absorb the new information technology and to use it to its maximal efficiency, just as it took time

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<sup>9</sup>According to Bergeron, Fauvel and Paquet (1995), Canada hit a recession from January 1975 to arch 1975, from May 1980 to June 1980, from August 1981 to November 1982, and from April 1990 to March 1991. We chose the breakpoints before the slump years 1975 and 1982 to compare productivity performances as much as possible over comparable phases of the business cycles.

to adjust to electricity at the beginning of the century. Second, the negative productivity growth is due to infrastructure, where the benefit is captured in the long run, but the short run contribution to the Solow residual is to the capital growth term, which has a minus sign indeed. Third, it is illuminating to diagnose the negative residuals at the sectoral level. A term by term inspection in the sense of equation (8) reveals that the slowdown in the second period is ascribed to the primary sector and that the last period shows negative technical change in communication and transportation. Technical regress in the primary sector is known to occur when minerals are not accounted (Carter, 1970) and a negative Solow residual in communication and transportation is another symptom of costs preceding benefits, as noted for infrastructure.

It is interesting to contrast our measure of technical change (Table 3, line 2) with the traditional Solow residual, which we have added to Table 3. The main distinction of our productivity measures is the endogeneity of value shares. Prices are marginal productivities and quantities reflect frontier allocations. The Solow residual is a Domar weighted average of sectoral productivity growth rates, see (9), but our Domar weights are different, say from Wolff (1985), by the use of competitive quantities and supporting prices for commodities and factor inputs. Table 3 reveals quite dramatic differences. The market-price based Solow residual is fairly unbiased in the period 1962-1974, but overstates the role of technical change in the periods 1974-1981 and 1981-1991. The terms-of-trade effect was far more important in explaining total factor productivity growth, particularly in the 1980s.

The intended contribution of this paper is to demonstrate that, at least in principle, productivity can be measured without recourse to factor shares or prices. The main reason for this disclaimer is that our model is fairly macro-economic in nature, featuring only one type of labor and three types of capital, with perfect mobility across sectors. More detailed specifications would affect the shadow prices and hence TFP. For example, if some type of capital is sector specific, then its constraint separates and each sector yields its own rate of return.

## 4 Conclusion

Standard measures of TFP-growth hinge on the use of value shares, hence of factor input prices. Since the latter are presumed to match factor productivities, the standard procedure amounts to accepting at face value what is supposed to be measured. In this paper we have demonstrated that factor productivities can be determined as the Lagrange multipliers to a program that maximizes the level of domestic final demand. The consequent measure of total factor productivity growth encompasses not only the Solow residual, but also terms-of-trade and preference-shift effects.

We have applied our new measure of TFP-growth to the Canadian economy in the period from 1962 to 1991. Canadian TFP grew by 2.6% yearly in the 1960s, dropped in the 1970s and recovered to 3.8% yearly in the 1980s. The bulk of its can be ascribed to labor productivity growth. Of the capital stock the infrastructure component is the main driving force. The healthy TFP-growth in the 1960s and the slowdown in the 1970s were both caused by technical change, but the recovery in the 1980s was due almost initially to an improvement in the terms-of-trade.

The Solow residual measures the shift of the production possibility frontier of an economy that is presumed to be on its frontier. When this assumption is not tenable, this paper shows how the frontier can be traced using input-output statistics. The Lagrange multipliers to the program that determines potential GDP measure the factor productivities.

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Table 1: Factor productivities (shadow prices)

Year	Labor	Buildings	Equipment	Infrastructure	Debt
1962	16.13	0.32	0.00	0.20	0.71
1963	16.50	0.33	0.00	0.19	0.71
1964	17.46	0.26	0.00	0.22	0.69
1965	17.86	0.28	0.00	0.18	0.69
1966	18.28	0.28	0.00	0.18	0.69
1967	19.31	0.20	0.00	0.18	0.68
1968	20.38	0.21	0.00	0.17	0.67
1969	20.91	0.17	0.00	0.18	0.67
1970	20.40	0.19	0.00	0.23	0.68
1971	21.78	0.14	0.00	0.24	0.66
1972	22.44	0.08	0.00	0.28	0.66
1973	22.96	0.05	0.00	0.32	0.65
1974	23.24	0.01	0.00	0.47	0.61
1975	22.70	0.05	0.00	0.45	0.64
1976	23.61	0.12	0.00	0.37	0.64
1977	24.52	0.08	0.00	0.34	0.65
1978	24.83	0.07	0.00	0.31	0.65
1979	24.85	0.06	0.00	0.32	0.65
1980	24.60	0.07	0.00	0.28	0.66
1981	24.31	0.10	0.01	0.25	0.69
1982	29.66	0.00	0.00	0.18	0.57
1983	12.07	0.62	0.83	0.15	0.82
1984	12.22	0.49	1.03	0.11	0.81
1985	23.11	0.24	0.22	0.16	0.73
1986	20.09	0.18	0.83	0.05	0.72
1987	20.76	0.11	0.99	0.03	0.70
1988	44.11	0.00	0.00	0.01	0.31
1989	22.41	0.00	1.21	0.00	0.63
1990	44.33	0.00	0.00	0.01	0.32
1991	46.13	0.00	0.00	0.01	0.29

Labor productivity is in 1986\$ per personhour. Capital (buildings, equipment and infrastructure) and debt productivities are rates of return.

Table 2: Productivity growth (annualized percentages) by factor input

	1962-1974	1974-1981	1981-1991
Total	2.6	-0.5	3.8
Labor	2.4	0.5	5.0
Capital	0.2	-1.0	-1.1
Buildings	-0.9	0.4	-0.3
Equipment	0.0	0.1	0.3
Infrastructure	1.1	-1.5	-1.2
Deficit	-0.0	0.0	-0.1

Table 3: Productivity growth (annualized percentages) by source of structural change

	1962-1974	1974-1981	1981-1991
Total	2.6	-0.5	3.8
Technical change	1.7	-1.3	-0.3
Terms-of-trade effect	0.7	0.5	3.8
Preference shift	0.2	0.3	0.2
Solow residual at market prices	1.4	0.5	0.2

## APPENDIX: Data

The constant price input-output tables obtained from Statistics Canada are expressed in 1961 prices from 1962 to 1971, in 1971 prices from 1971 to 1981, in 1981 prices from 1981 to 1986, and in 1986 prices from 1986 to 1991. All tables have been converted to 1986 prices using the chain rule. For reasons of confidentiality, the tables contain missing cells, which we have filled using the following procedure. The vertical and horizontal sums in the make and use tables are compared with the reported line and column totals, which do contain the missing values. We select the rows and columns where the two figures differ by more than 5% from the reported totals, or where the difference exceeds \$250 million. We then fill holes or adjust cells on a case by case basis filling in priority the intersections of the selected rows and columns, using the information on the input or output structure from other years, and making sure the new computed totals do not exceed the reported ones.

There are three capital types, namely buildings, equipment, and infrastructure.<sup>10</sup> The gross capital stock, hours worked and labor earnings are from the KLEMS database of Statistics Canada, described in Johnson (1994). In particular, corrections have been made to include in labor the earnings of the self-employed, and to separate business and non-business labor and capital. The total labor force figures are taken from Cansim (D767870) and converted in hours using the number of weekly hours worked in manufacturing (where it is the highest). Out of the 50 industries, no labor nor capital stock data exist for sectors 39, 40, 48, 49, 50, and no capital stock data for industry 46. The capital stock for industry 46 has been constructed using the capital/labor ratio of industry 47 (both industries producing predominantly the same commodity).

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<sup>10</sup>Statistics Canada calls them "building constructions," "equipment" and "engineering constructions." Alternatively we could have modeled capital as being sector-specific, the so called putty-clay model. We prefer the present hypothesis of sectoral mobility of capital within each group for three reasons. First, to let the economy expand, we would have needed capacity utilization rates which are badly measured and unavailable for a number of service sectors. Second, to relieve a numerical collinearity problem, we would have to relieve the capital constraint on the non-business sector. Third, the combination of 11 non-tradeables and sector-specific capacity expansion limits is too stringent. It would lead to a high shadow price on construction commodities and zero shadow prices almost anywhere else.

The international commodity prices are approximated by the U.S. prices, given that 70% of Canada's trade is with the United States. We have used the U.S. producer prices from the U.S. Bureau of Labor Statistics, Office of Employment Projection. The 169 commodity classification has been bridged to Statistics Canada's 94 commodity classification. As the debt constraint in (1) is given in Canadian dollars, we convert U.S. prices to Canadian equivalents. We have used, whenever available, unit value ratios, (UVRs, which are industry specific) computed and kindly provided to us by Gjalt de Jong (1996). The UVRs are computed using Canadian quantities valued at U.S. prices. For the other commodities, we have used the purchasing power parities computed by the OECD (which are based on final demand categories). The UVRs establish international price linkages for 1987, the PPPs for 1990 in terms of Canadian dollars per U.S. dollar. We hence need two more transformations. First, U.S. dollars are converted to Canadian dollars using the exchange rates taken from Cansim (series 0926/133400). Second, since the input-output data are in 1986 prices, we need the linkage for 1986, which is computed by using the respective countries' commodity deflators: the producer price index for the U.S. (see above) and the total commodity deflator from the make table (except for commodities 27, 93 and 94, for which we use the import deflator from the final demand table) for Canada. Finally, international commodity prices are divided by a Canadian final demand weighted average of international commodity prices to express them in real terms.

Are considered as non-tradeable, services incidental to mining, residential construction, non-residential construction, repair construction, retail margins, imputed rent from owner occupied dwellings, accommodation & food services, supplies for office, laboratories & cafeteria, and travel, advertising & promotion, for which no trade shows up in the input-output tables for most of the sample period.

The structure of some non-tradeability constraints implies the equality of the activity levels of "construction" and final demand, "owner-occupied dwellings" and final demand, and "printing and publishing" and "travel, advertising and promotion." We have forced the activity level of industry 39 (government royalties on natural resources, which essentially pertains to oil drilling in Alberta) to follow industry 5 (crude petroleum and natural gas) to ensure there are no such royalties without oil drilling. A more detailed documen-

tation of the data and their construction is available from the authors upon request.

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