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# **Can Financial Intermediation Induce Economic Fluctuations?**

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# Can Financial Intermediation Induce Economic Fluctuations?\*

Sanjay Banerji<sup>†</sup>, Ngo Van Long<sup>‡</sup>

## Résumé / Abstract

On étudie un modèle qui montre que l'intermédiation active des institutions financières peut générer les fluctuations. Il s'agit d'un modèle aux générations imbriquées avec un stock de capital. Les individus sont riscophobes, tandis que les institutions financières (I.F.) ne le sont pas. On considère deux cas. Dans le premier cas, les I.F. sont actives : elles prêtent de l'argent sous la condition que les emprunteurs acceptent des restrictions sur leurs choix de projets d'investissement. Dans le deuxième cas, les I.F. sont passives. Nous démontrons que si les I.F. sont actives, les conditions de prêts peuvent créer un effet de richesse qui peut générer les fluctuations du taux d'investissement, et du P.I.B.

*We construct a model to show that active financial intermediation can induce economic fluctuations. We embed a financial sector in a simple overlapping generation model with a single stock of capital. Individuals are risk averse agents that face idiosyncratic risks in their business activities: Due to limited liability, agents have incentives to invest in a technology that produces high output with a smaller probability. Financial intermediaries (FIs) are risk neutral. We distinguish two scenarios. The first scenario is one with “active financial intermediation”: the FIs lend only on the conditions that borrowers accept restrictions on their investments. In the second scenario, financial intermediation is “passive”, in that the FIs lend without monitoring the activities of the borrowers. For a given loan size, the investment level under active financial intermediation is shown to be smaller than under passive financial intermediation. This fact alone creates, in the first scenario, an income effect that may generate fluctuations in investment. (This effect is absent under passive financial intermediation, and, as a result, in our model there are no fluctuations under passive financial intermediation.) Thus business cycles and possibly chaotic dynamics can be, under certain conditions, generated by active intermediation.*

**Mots Clés :** Intermédiation financière, fluctuations endogènes

**Keywords:** Financial intermediation, endogenous fluctuations

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# 1 Introduction

Monitoring the degree of risks of the projects undertaken by borrowers and providing insurance against idiosyncratic shocks to depositors are two primary functions performed by financial intermediaries (FIs). The monitoring role of the FIs primarily stems from the limited liability clause embedded in debt contracts. The clause stipulates that in the event of default, borrowers' obligations are limited. Since borrowers' interests in the event of default are protected, they tend to undertake excessively risky project, giving rise to potential conflicts with lenders regarding the choice of project risk. The FIs tend to counteract such moves by resorting to capital rationing. Many authors have investigated this phenomenon in a partial equilibrium context. An equally important role of financial intermediary is to provide assets to its depositors that shield income against idiosyncratic shocks. In an environment where risk-averse agents face idiosyncratic income shocks, it is plausible to think that FIs would naturally arise to provide efficient risk-sharing. After all, a well-diversified intermediary, assured by the law of large numbers, would achieve complete risk sharing in such an environment.<sup>1</sup> Hence, reduction of the project risks undertaken by the borrowers through the restrictions in the use of capital as well as provision of loan that helps agents smooth out consumption across different states of nature are the twin objectives of financial intermediaries.

In this paper, we show that fulfilment of both objectives (capital rationing and consumption loans) can expose the economy to non-linear fluctuations, including chaos and cycles. The intuitive reasoning is as follows: Suppose that the technology available to individuals for converting an investment into a flow of final goods is risky, and they can affect the risk by the choice of investment. In particular, if a higher investment level is associated with a greater level of output but with a lower chance of success, there is a potential conflict between lenders and borrowers, given that the project is financed by debt. In this type of situation, agents tend to undertake a risky project which generates a very high output with a smaller probability because in the event of success, they can keep the surplus while in the event of failure, loss is minimal due to limited liability. If investment level can be observed

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<sup>1</sup>It has been argued that the FIs perform several other important functions, such as delegated monitoring, etc. A fuller description of these functions is available in Pagano (1993). In this paper, we focus on the risk-sharing role of financial intermediaries.

(via monitoring), financial intermediaries may want to restrict the size of investment so that risk is reduced. On the other hand, provision of insurance requires disbursement of loans so that consumption is fully smoothed out across different states of nature. These two features imply that the amount of loan that agents will receive for smoothing out consumption will exceed the amount of investment in the risky projects that they undertake. We will show that this creates a wealth effect and can generate non-linear fluctuations in output over time via its effect on savings.

The source of fluctuations in our model is the income effect (or wealth effect) that is induced by the optimal contracts. This is distinct from a more well-known source of fluctuations that has been discussed extensively in the literature on chaos, namely, the structure of preferences. Suppose there is a fall in the aggregate capital stock. Agents will receive a smaller wage and this will tend to depress savings and investment. On the other hand, such a fall in the capital stock will enhance the probability of success and will lead to a lower borrowing rate and disbursement of a higher volume of loans. The second effect is the wealth effect (captured in lemma 2 below) and we show below that only the existence of an active financial intermediation regime can generate such effects that usher in non-linear fluctuations in capital stock and output.

The economy we study is similar in some respects to the one explored in Azariadis (1993). In particular, the setting is a production economy inhabited by overlapping generations of risk-averse agents. There is a single final good produced according to a standard neoclassical production function using capital and labor as inputs. In contrast to the specifications of the standard neoclassical growth model, however, there is one major modification. Here, at the micro-level, individuals face idiosyncratic risks, represented by two states: success or failure. The probability distribution of the uncertain future output generated by an investment project depends on the amount of capital invested by the agent. Capital then has a dual role: on the one hand, more capital investment decreases the probability of success with the output technology (henceforth “project”), while on the other hand, through standard channels, more capital means more output in the success state.

We distinguish between two types of intermediation regimes: passive and active. A passive intermediation regime is one where intermediaries lend indiscriminately and do not interfere in any way with the activities of the borrowers. In contrast, an active intermediation regime is characterized by intermediaries that lend only to borrowers who accept restrictions on their

own actions as stipulated by the intermediary within the context of a contractual arrangement. This distinction is entirely analogous to and reminiscent of the difference between publicly-placed debt and bank debt as discussed in Rajan (1992).<sup>2</sup> In the public-debt market, there are numerous creditors; consequently, no single creditor has any incentive to monitor the borrower (“passive intermediation”). Public debt therefore is an “arms-length” contractual relationship. On the other hand, in the private-debt market, a bank or a consortium of banks, being a single party, has a bigger stake in the lending. Hence, lending agreements typically contain loan covenants which are contractual restrictions imposed on the behaviour of the borrowing firm. These restrictive covenants (“active intermediation”) induce a borrower to undertake potentially costly actions which would otherwise not have been taken had the decision been left solely to the borrower. Such a distinction is routine in the corporate finance literature.

In both intermediation regimes, agents seek to borrow funds in order to make capital investments on their projects. At the same time, they set aside some wealth in the form of deposits with financial intermediaries which promise them a safe default-free rate of return. In this complete information world, contractual credit arrangements are such that complete risk-sharing is achieved; i.e., all agents enjoy a non-stochastic consumption stream over their lifetimes. We show that in the passive intermediation regime, an agent will, of his own accord, invest in his idiosyncratic risky project the entire amount of the loan he receives. His capital investment is therefore completely financed by outside public debts. In sharp contrast, in an active intermediation regime, efficient risk sharing will require that an agent invest less than the amount of loan he receives. Thus, for a given loan size, in an active intermediation regime (private debt regime), agents are required to invest less than in the passive regime.

We will establish that, in the context of our model, in a passive intermediation regime, the law of motion for the capital stock is strictly monotonic, while in an active intermediation regime, the law of motion for the capital stock *may* be nonmonotonic, with the possibility of endogenous cyclical fluctuations and chaotic dynamics. Put differently, economies with a passive intermediation regime are immune to endogenous fluctuations while those with an active intermediation regime are not. In the latter case, the ratio of

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<sup>2</sup>While Rajan (1992) focuses on the costs of bank debt in an asymmetric information environment, ours is a complete information setting with idiosyncratic risks.

internal funds to total capital investment may also exhibit cyclical fluctuations.

The possibility that financial intermediation in and of itself may expose an economy to endogenous fluctuations has been recognized earlier in the macroeconomics literature. Friedman (1960) in fact advocated 100% reserve requirements on financial intermediaries as a sure step towards eliminating such fluctuations. Smith (1991) and Woodford (1990) have examined the validity of Friedman's proposal in a world with financial intermediaries and nominal assets. Schreft and Smith (1997) establish the existence of complex dynamics within the context of a *monetary* growth model with active financial intermediaries. Allen and Gale (1997) consider the role of financial intermediary in an overlapping generation framework where markets are not complete. In their model, financial markets help agents to smooth out consumption across time; and they focussed on welfare comparisons between financial markets and financial intermediaries. In our set-up, while both passive financial markets and active financial intermediation help agents to smooth out idiosyncratic shocks, it is active intermediation that leads to endogenous fluctuations of economic activity. To the best of our knowledge, the issue of whether financial intermediation can induce *real* endogenous cyclical fluctuations has not been explored in the literature.

It is also important to emphasize that these cycles emerge in a relatively standard economy; in particular, we do not rely on factors such as limited market participation, imperfect competition, multiple sectors etc., which have been shown to contribute to cyclical fluctuations (see Boldrin and Woodford (1990) for a survey). We do however rely on of the "income effect" (or more appropriately, the "internal equity effect" in our case) in the generation of cyclical fluctuations, although it should be noted that the nature of the income effect we describe is very different from the one explored in Azariadis (1981) or Grandmont (1985), who both stipulated restrictions on preferences so as to generate a sufficiently strong income effect which in turn, produces "backward-bending" savings/labor-supply functions. In our set-up, however, the income effect is endogenously generated. Active intermediaries require agents to invest less than the loan they provide for smoothing out consumption. This is the source of the income (internal equity) effect.<sup>3</sup>

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<sup>3</sup>The *strength* of this effect depends on the curvature properties of the probability density function describing future uncertain output.

## 2 The Model

There is a continuum of entrepreneurs, which we represent by the real interval  $[0, 1]$ . The measure of this continuum is 1. The generic entrepreneur is denoted by the index  $\theta \in [0, 1]$ . There is also a continuum of workers, with measure 1. Each worker inelastically supplies a fixed amount of labor. The total supply of labor by the continuum of workers is assumed to be 1.

An entrepreneur who plans to produce in period  $t + 1$  must invest in period  $t$ . The amount that entrepreneur  $\theta$  invests in period  $t$  is denoted by  $K_{t+1}(\theta)$ . The outcome of the investment is subject to a random event, which is idiosyncratic. (One may think of the project as an R&D activity). At the beginning period  $t + 1$ , the uncertainty is resolved, and the entrepreneur then knows whether his investment project is a success or a failure. If it is a failure, the amount invested,  $K_{t+1}(\theta)$ , becomes useless, and he will not hire any worker. If it is a success, he will go to the labor market to hire labor at the given wage rate  $W_{t+1}$ , and his firm's output is

$$Y_{t+1}(\theta) = F(K_{t+1}(\theta), N_{t+1}(\theta))$$

where  $N_{t+1}(\theta)$  is the amount of labor hired, and  $F(., .)$  is a neoclassical production function, with constant returns to scale. Given  $K_{t+1}(\theta)$ , the owner of the successful project will choose  $N_{t+1}(\theta)$  to maximize his profit, by equating marginal product of labor,  $F_N$ , to the wage rate  $W_{t+1}$ . This implies that his firm's capital labor ratio, denoted by  $x_{t+1}(\theta) = K_{t+1}(\theta)/N_{t+1}(\theta)$ , satisfies the equation

$$f(x_{t+1}(\theta)) - f'(x_{t+1}(\theta))x_{t+1}(\theta) = W_{t+1} \quad (1)$$

where  $f(x_{t+1}(\theta)) \equiv F(x_{t+1}(\theta), 1)$ . Equation (1) yields  $x_{t+1}(\theta)$  as a function of  $W_{t+1}$ . We denote this function by  $\phi(.)$  :

$$x_{t+1}(\theta) = \phi(W_{t+1}) \quad (2)$$

where  $\phi'(\cdot) > 0$ . After making payments to labour, the successful entrepreneur gets the residual  $\Pi$ , where

$$\Pi \equiv f'(\phi(W_{t+1}))K_{t+1}(\theta). \quad (3)$$

For example, if  $F = K^\alpha N^{1-\alpha}$ , then

$$x_{t+1}(\theta) = \left[ \frac{W_{t+1}}{1-\alpha} \right]^{1/\alpha} = \phi(W_{t+1}) \quad (4)$$



Inverting, we get

$$W_{t+1} = x_{t+1}^\alpha (1 - \alpha).$$

The probability that entrepreneur  $\theta$  is successful is assumed to be dependent on the amount he invests, and is denoted by  $p(K_{t+1}(\theta))$ . The probability that he is unsuccessful is  $1 - p(K_{t+1}(\theta))$ . We assume that  $1 \geq p(0) > 0$  and that  $p(\cdot)$  has **negative** derivative:  $p'(K_{t+1}) < 0$ . This assumption on  $p'$  is analogous to that made by Stiglitz and Weiss (1981, 399-400), and de Meza and Webb (1999, 135). Basically, our assumption states that a higher  $K_{t+1}$  implies a greater total return, if the project is successful, as indicated by (3), but also a higher probability that the project will be unsuccessful. Large investment projects usually involve higher degrees of complexity, and therefore a greater probability of failure. (We could have introduced a risk-characteristic parameter  $\omega$  and let  $p = p(\omega)$  and let output in the success state be an increasing function of  $\omega$ ; the added complexity is not worthwhile, because basically our essential results remain unchanged.)

In what follows, we assume that all entrepreneurs choose the same level of investment, that is  $K_{t+1}(\theta) = K_{t+1}(\theta')$  for all  $\theta$  and all  $\theta'$ . (We will thus drop the index  $\theta$  when there is no ambiguity.) Even though there is uncertainty at the individual level, in the sense that no entrepreneur knows in advance whether he will be successful, we take it that there is complete certainty in the aggregate. By this, we mean that if all entrepreneurs invest the same amount  $K_{t+1}$ , then the measure of successful entrepreneurs (i.e., the fraction of firms that are successful) is exactly  $p(K_{t+1})$ , and therefore, after individual entrepreneurs observe their idiosyncratic random events, the economy's *overall capital-labor ratio* is  $p(K_{t+1})K_{t+1}/1$  (recall that  $p(K_{t+1})$  is the fraction of firms that survive, and the measure of labor is 1.) It follows that for the successful entrepreneur, his firm's capital labor ratio is

$$x_{t+1} = p(K_{t+1})K_{t+1}/1 \tag{5}$$

From equations (2) and (5), we obtain

$$p(K_{t+1})K_{t+1} = \phi(W_{t+1}) \tag{6}$$

This equation tells us that, given the investment level  $K_{t+1}$  undertaken by all entrepreneurs, the market-clearing wage rate in period  $t + 1$  is uniquely determined.

We now turn to the problem of how investment decisions are made. This depends on nature of the credit market, and on the entrepreneur's utility function.

We assume that each individual's life span consists of two periods. In the first period, he supplies one unit of labor, and earns the wage income  $W_t$ . He can borrow from the financial intermediaries (FIs) an amount  $B_t$ , and consume the amount  $C_t^y$  (where the superscript  $y$  indicates consumption when the consumer is young). The difference  $(W_t + B_t) - C_t^y$  is carried over to the next period, by (a) investing an amount  $K_{t+1}$  in his own firm (a risky activity), and (b) investing an amount  $S_t$  in a safe asset (such as a bank deposit insured by the government.) Thus the first period budget constraint is

$$C_t^y = (W_t + B_t) - K_{t+1} - S_t.$$

By investing  $S_t$  in the riskless asset, in period  $t + 1$  he gets back  $r_{t+1}S_t$ , where  $r_{t+1}$  is  $1 + i_t$ , and  $i_t$  is the net rate of interest on the safe asset. We call  $r_{t+1}$  the gross rate of return on the safe asset, or simply the "safe rate of return". (We will show later that this rate is endogenously determined.) The investment in the risky activity  $K_{t+1}$  may turn out to be a success, or a failure, as mentioned above.

If the risky project turns out to be a success, (i.e., the state of nature is "good") the entrepreneur will have to pay to the FIs the contracted amount  $R_{t+1}B_t$  where  $R_{t+1}$  is the gross rate of interest on the risky loan. If the project is unsuccessful, (i.e., in the bad state) he will pay the FIs nothing (this reflects the limited liability feature of the loan contract). We will soon discuss how  $R_{t+1}$  is determined.

The individual's second period consumption in the "good state" is

$$C_{t+1}^{oG} = r_{t+1}S_t + f'(\phi(W_{t+1}))K_{t+1} - R_{t+1}B_t$$

(where  $o$  indicates the old age consumption, and  $G$  indicates the good state). In the "bad state",

$$C_{t+1}^{oB} = r_{t+1}S_t$$

because in the bad state, his firm is a complete failure, and under limited liability, he pays the FIs nothing.

Note that if  $W_t, W_{t+1}, r_{t+1}, K_{t+1}, R_{t+1}$  and  $B_t$  are given, the consumer's choice of  $S_t$  must maximize

$$\begin{aligned} &U_1 [W_t + B_t - K_{t+1} - S_t] + [1 - p(K_{t+1})] V_2 [r_{t+1}S_t] \\ &+ p(K_{t+1}) V_2 [r_{t+1}S_t + f'(\phi(W_{t+1}))K_{t+1} - R_{t+1}B_t] \end{aligned}$$

where  $U_1(\cdot)$  is the utility function for the first period, and  $V_2(\cdot)$  is the utility function for the second period. This maximization problem yields the function

$$S_t^* = S[W_t, W_{t+1}, r_{t+1}, K_{t+1}, R_{t+1}, B_t].$$

### 3 Financial contracts under active intermediation

Financial intermediaries (FIs) pay depositors the safe rate  $r_{t+1}$  (in period  $t+1$ ) for each dollar they borrows from depositors in period  $t$ . The FIs lend to entrepreneurs, and the expected return from each dollar lent is  $p(K_{t+1})R_{t+1}$ , where  $K_{t+1}$  is the amount invested by the representative entrepreneur. The FIs are perfectly competitive, and risk neutral. Zero expected profit for the FIs implies

$$p(K_{t+1})R_{t+1} = r_{t+1}$$

Now the FIs must design the loan contract that they offer to the entrepreneurs. In this section, we consider the case of “*active intermediation*”: the FI that lends to entrepreneur  $\theta$  actively monitors the entrepreneur, in the sense that it *observes* the amount  $K_{t+1}(\theta)$  that entrepreneur  $\theta$  invests in the risky project. The efficient contract must maximize the expected utility of the entrepreneur, given the limited liability constraint, and subject to zero expected profits for the FIs. It is as if the FIs were to tell the entrepreneurs how much they should borrow, how much to invest in the risky asset, how much to invest in the safe asset, and how much to consume in each state.

Formally, given  $W_t, W_{t+1}, r_{t+1}$ , the design of the efficient contract solves the following problem: choose  $S_t(\theta), K_{t+1}(\theta), B_t(\theta), R_{t+1}(\theta)$  to maximize entrepreneur  $\theta$ 's expected utility

$$\begin{aligned} EU(\theta) = & U_1 [W_t(\theta) + B_t(\theta) - K_{t+1}(\theta) - S_t(\theta)] + [1 - p(K_{t+1}(\theta))] V_2 [r_{t+1} S_t(\theta)] \\ & + p(K_{t+1}(\theta)) V_2 [r_{t+1} S_t(\theta) + f'(\phi(W_{t+1})) K_{t+1}(\theta) - R_{t+1}(\theta) B_t(\theta)] \end{aligned}$$

subject to

$$p(K_{t+1}(\theta)) R_{t+1}(\theta) = r_{t+1} \tag{7}$$

We will omit the index  $\theta$  in what follows, to lighten notation. Write the Lagrangian

$$L = EU + \lambda [p(K_{t+1}) R_{t+1} - r_{t+1}]$$

The first order conditions are:

(i) w.r.t.  $S_t$

$$-U'_1(C_t) + rpV'_2 [C_{t+1}^{oG}] + r(1-p)V'_2 [C_{t+1}^{oB}] = 0 \quad (8)$$

(ii) w.r.t.  $K_{t+1}$

$$-U'_1(C_t) + p'V_2 [C_{t+1}^{oG}] + pV'_2 [C_{t+1}^{oG}] f'(\phi(W_{t+1})) - p'V_2 [C_{t+1}^{oB}] + \lambda p' R_{t+1} = 0 \quad (9)$$

(iii) w.r.t.  $B_t$

$$U'_1 + R_{t+1}pV'_2 [C_{t+1}^{oG}] = 0 \quad (10)$$

(iv) w.r.t.  $R_{t+1}$

$$pV'_2 [C_{t+1}^{oG}] B_t - \lambda p = 0 \quad (11)$$

From (9), (10) and (7):

$$rpV'_2 [C_{t+1}^{oG}] + r(1-p)V'_2 [C_{t+1}^{oB}] = R_{t+1}pV'_2 [C_{t+1}^{oG}] = rV'_2 [C_{t+1}^{oG}]$$

It follows that  $V'_2 [C_{t+1}^{oG}] = V'_2 [C_{t+1}^{oB}]$ , hence

$$C_{t+1}^{oG} = C_{t+1}^{oB}. \quad (12)$$

In other words, second period consumption is independent of the state of nature. This is a standard result: the consumer is given complete insurance.

We state this result as lemma 1:

**Lemma 1:** The consumer is provided with full insurance.

**Lemma 2:** The efficient contract implies that for each entrepreneur, the ratio of the loan to the amount of capital invested in the risky project is equal to the ratio of (ex-post) marginal product of capital to the gross interest rate on the risky loan:

$$\frac{B_t}{K_{t+1}} = \frac{f'(\phi(W_{t+1}))}{R_{t+1}} \quad (13)$$

**Proof:** Use (12). ■

We now characterize the relationship between the safe rate of return  $r_{t+1}$  and the expected marginal product of capital. To this end, let us define the elasticity of success probability with respect to capital, which we call “success elasticity” for short:

$$\eta(K) \equiv \frac{Kp'(K)}{p(K)}$$

**Assumption A1:** The “success elasticity” lies within the open interval  $(-1, 0)$  for all  $K > 0$  :

$$-1 < \eta(K) < 0.$$

**Remark:** Assumption A1 implies that  $1 + \eta(K_{t+1}) > 0$  for all  $K > 0$ . As  $K$  tends to zero,  $p'(K)K$  tends to zero if  $p'(0)$  is finite. If  $\eta(0) = 0$ , assumption A1 implies that  $\eta'(0) > 0$ .

As an example, consider the function

$$p(K) = \frac{\beta}{1 + K} \tag{14}$$

then

$$0 > \eta(K) = -\frac{K}{1 + K} > -1$$

Thus Assumption A1 is satisfied.

We obtain the following lemma:

**Lemma 3:** Given  $W_{t+1}$  and the safe rate of return  $r_{t+1}$ , the efficient contract specifies a unique investment amount  $K_{t+1}$ . This amount equates the expected rate of return, modified by the success elasticity, to the safe rate of return:

$$r_{t+1} = p(K_{t+1})f'(\phi(W_{t+1})) [1 + \eta(K_{t+1})] \tag{15}$$

Thus, under assumption A1, *the safe rate of return is smaller than the expected rate of return.*

**Proof:** From (12), (9), (10) and (11)

$$r_{t+1} = pf'(\phi(W_{t+1})) + B_t R_{t+1} p' \tag{16}$$

From (13) and (16), we get

$$r_{t+1} = pf' + Kp'f' \tag{17}$$

which gives (15) ■

Notice that, under Assumption A1, the safe rate of return  $r_{t+1}$  is positive.

**Lemma 4:** Regardless of the values of  $W_{t+1}$  and of  $r_{t+1}$ , in an efficient contract, the ratio of investment in the risky project to the the amount of loan is always equal to 1 plus the success elasticity.

**Proof:** from (15) and (7),

$$R_{t+1} = f'(\phi(W_{t+1})) [1 + \eta(K_{t+1})]$$

This equation and (13) give

$$\frac{K_{t+1}}{B_t} = 1 + \eta(K_{t+1}) \quad (18)$$

**Remark:** Since  $-1 < \eta < 1$ , (18) implies  $B_t > K_{t+1}$  if  $K_t > 0$ . This may seem at first surprising: The FIs lend to the entrepreneur more than what he actually “needs” for the risky investment project<sup>4</sup>. Upon reflection, the excess of  $B_t$  over  $K_{t+1}$  being used to invest in the safe asset, this ensures that the second period consumption of an entrepreneur in the event of failure is greater than the second period consumption he would have if he were to choose not to be an entrepreneur. (Note that because of the assumption of full observability, there is *no moral hazard* here.)

Condition (18) implies that  $B_t$  can be expressed as a function of  $K_{t+1}$ :

$$B_t = B_t(K_{t+1}) = \frac{K_{t+1}}{1 + \eta(K_{t+1})} \quad (19)$$

Define

$$Z_t = B_t(K_{t+1}) - K_{t+1} \equiv Z(K_{t+1}) \quad (20)$$

The variable ( $Z_t$ ) plays a key role in our set-up because it is the source of the income effect that causes endogenous cycles and fluctuations. We will show below that this effect is operative whenever intermediaries observe and control  $K$  but not otherwise. Hence, the source of non-linear fluctuations can be attributed to financial contracts written with an active intermediary.

Let us now turn to the amount invested in the safe asset. Making use of (20) and lemmas 1 to 4 above, we can simplify condition (8) as

$$-U'_1 [W_t + Z_t - S_t] + r_{t+1} V'_2 [r_{t+1} S_t] = 0 \quad (21)$$

This equation yields  $S_t$  as a function of  $W_t$ ,  $r_{t+1}$  and  $Z_t$

$$S_t = S [W_t, r_{t+1}, Z_t]$$

with derivatives

$$\frac{\partial S}{\partial W_t} = \frac{1}{\Delta} U''_1 > 0, \quad \Delta \equiv U''_1 + (r_{t+1})^2 V''_2 < 0,$$

---

<sup>4</sup>This is due the assumption that  $p' < 0$ : the more an entrepreneur invests, the greater is the probability of failure, even though the return is greater if he succeeds. (Note the importance of the assumption that that  $p' < 0$ : If, on the contrary,  $p' > 0$ , then we would have  $B_t < K_{t+1}$ .)

$$\frac{\partial S}{\partial Z_t} = \frac{1}{\Delta} U_1'' > 0$$

and

$$\frac{\partial S}{\partial r_t} = -\frac{1}{\Delta} [V_2' + C_{t+1}^o V_2'']$$

which is positive, zero, or negative, according to whether the second period elasticity of marginal utility,  $-C_{t+1} V_2''/V_2'$  is smaller than, equal to, or greater than unity.

## 4 Dynamics under active financial intermediation

We are now ready to study the dynamics of the model. We require that the financial market be in equilibrium, therefore the deposits  $S_t$  in the FIs must be equal to the loans the FIs make to entrepreneurs:

$$S [W_t, r_{t+1}, Z_t] = B_t \quad (22)$$

We now express the left-hand side of (22) as a function of  $K_t$  and  $K_{t+1}$  and its right-hand side as a function of  $K_{t+1}$ . This will give us a first order difference equation in  $K$ , from which we can examine stationary states and their stability properties.

From (6) and (1),

$$W_t = \phi'^{-1} [p(K_t)K_t] = f [p(K_t)K_t] - f' [p(K_t)K_t] [p(K_t)K_t] \equiv \psi(K_t) \quad (23)$$

(Note that if (4) is assumed, then  $\psi(K_t) = (1 - \alpha) [p(K_{t+1})K_{t+1}]^\alpha$  and  $\psi'(K_t) = (1 - \alpha) [p(K_{t+1})K_{t+1}]^{\alpha-1} [p'(K_t)K_t + p(K_{t+1})] > 0$  by Assumption A1.)

From (15) and (6),

$$r_{t+1} = p(K_{t+1})f' [p(K_{t+1})K_{t+1}] [1 + \eta(K_{t+1})] \equiv \xi(K_{t+1}) \quad (24)$$

Thus (22) can be written as

$$S [\psi(K_t), \xi(K_{t+1}), Z(K_{t+1})] = B_t(K_{t+1}) \quad (25)$$

This first order difference equation constitutes the dynamics of the model.

If  $U_1'(0) = V_2'(0) = \infty$ , then  $K = 0$  is a steady state. (Note that  $B_t(0) = 0$ ,  $\psi(0) = 0$ , and  $Z(0) = 0$ ). With  $K = 0$ , there is no output, no income, and zero investment in the safe asset. There may exist several interior steady states. From (25) we obtain the slope

$$\frac{dK_{t+1}}{dK_t} = \frac{-S_W \psi'(K_t)}{S_Z [B'(K_{t+1}) - 1] + S_r \xi'(K_{t+1}) - B'(K_{t+1})} \quad (26)$$

where, due to Assumption A1,

$$\psi'(K_t) = \frac{dW_t}{dx_t} \frac{dx_t}{dK_t} = \frac{1}{\phi'} [p'(K_t)K_t + p(K_t)] > 0$$

Thus the numerator of the right-hand side of (26) is positive. The denominator is ambiguous in sign, because<sup>5</sup>

$$B'(K_{t+1}) = \frac{1}{[1 + \eta(K_{t+1})]^2} [(1 + \eta) - K_{t+1}\eta'] \quad (27)$$

$$\begin{aligned} \xi'(K_{t+1}) = & p'(K_{t+1})f' [p(K_{t+1})K_{t+1}] [1 + \eta(K_{t+1})] + \\ & p(K_{t+1})f'' [p(K_{t+1})K_{t+1}] [p'(K_t)K_t + p(K_t)] [1 + \eta(K_{t+1})] + \\ & p(K_{t+1})f' [p(K_{t+1})K_{t+1}] \eta' \end{aligned} \quad (28)$$

The first two terms on the right-hand side of (28) are negative, and, whenever  $\eta' < 0$ , the third term is also negative. On the other hand, if  $\eta' > 0$ , then we cannot determine the sign of  $\xi'(K_{t+1})$ . And even if  $S_r = 0$ , the denominator of the right-hand side of (26) is ambiguous in sign. It follows that there may exist points at which the denominator of (26) changes sign, i.e., the locus of points  $(K_t, K_{t+1})$  that satisfy (25) may well be a correspondence<sup>6</sup>  $K_{t+1} = \chi(K_t)$ : for a given  $K_t$  there may exist more than one values of  $K_{t+1}$ .

If  $K_{t+1} = \chi(K_t)$  is a correspondence and the denominator of (26) changes sign only once, then, in view of the fact that the numerator of (26) does not

<sup>5</sup>Since we have assumed that, for positive  $K$ ,  $1 > 1 + \eta > 0$ , it follows that  $B$  is always positive and greater than  $K$  for all  $K > 0$ . Thus it is not possible to have and  $B'(K_{t+1}) < 0$  for all  $K$ ; nevertheless, it is possible that  $B'(K_{t+1}) < 0$  over some intervals where  $\eta'$  is positive and sufficiently great.)

<sup>6</sup>If  $\chi(\cdot)$  is in fact a correspondence, the system is said to exhibit “indeterminacy”: for a given  $K_t$ , the next period capital stock  $K_{t+1}$  may take on several values, depending on the expectation about  $W_{t+1}$ . Multiple equilibria and self-fulfilling expectations are features of many economic models (see, for example, Roger E. A. Farmer’s book, “Macroeconomics of self-fulfilling prophecies”, 1999).



change sign, we can construct a U-shaped curve  $K_t = G(K_{t+1})$ . Then  $G(\cdot)$  represents the backward dynamics of this economy.

**Lemma 5:** The curve  $K_t = G(K_{t+1})$  changes sign only if there exists a point where

$$S_Z [B'(K_{t+1}) - 1] + S_r \xi'(K_{t+1}) - B'(K_{t+1}) = 0 \quad (29)$$

**Remark:** In the case  $S_r = 0$ , condition (29) holds if and only if there exists a point  $K_{t+1}^C$  such that

$$B'(K_{t+1}^C) = -\frac{U_1''}{[\xi(K_{t+1}^C)]^2 V_2''} \quad (30)$$

This is possible only if  $B'(K_{t+1}^C) < 0$  (i.e.,  $K_{t+1}^C \eta'(K_{t+1}^C) > 1 + \eta(K_{t+1}^C) > 0$ ).

**Example 1:** Let  $U_1 = aC_t^y - (b/2)^2 (C_t^y)^2$  and  $V_2 = \ln C_{t+1}^o (= \ln(r_{t+1} S_t))$ . Then  $S_r = 0$  and, in view of the equilibrium condition (25), the condition (30) is simply:

$$B'(K_{t+1}^C) = -bB(K_{t+1}^C)$$

(recall that  $S_t = B_t$  in equilibrium.)

**Example 2:** Recall the first order condition

$$-U_1' [W_t + B_t - K_{t+1} - S_t] + r_{t+1} V_2' [r_{t+1} S_t] = 0 \quad (31)$$

and the market clearing condition

$$S_t = B_t(K_{t+1}) \quad (32)$$

Instead of solving (31) for  $S_t = S[W_t, r_{t+1}, Z_t]$  and substituting into (32), we can substitute (32) into (31). This gives

$$U_1' [W_t - K_{t+1}] = r_{t+1} V_2' [r_{t+1} B_t(K_{t+1})]$$

hence

$$U_1' [\psi(K_t) - K_{t+1}] = \xi(K_{t+1}) V_2' [\xi(K_{t+1}) B_t(K_{t+1})] \quad (33)$$

which yields the first order difference equation for our system. If  $U_1 = \ln C^y$  and  $V_2 = \ln C^o$ , then the difference equation (33) simplifies to

$$\psi(K_t) = K_{t+1} + B_t(K_{t+1}) \equiv \gamma(K_{t+1}) = \left[ \frac{2 + \eta}{1 + \eta} \right] K_{t+1} \quad (34)$$

(It is easy to verify that, if  $p(K)K$  tends to zero as  $K$  tends to zero, and if  $W$  tends to zero as  $x$  tends to zero, then (34) has  $(0, 0)$  as a stationary point.) Since  $\psi(K_t)$  is monotone increasing in  $K_t$  and  $Z(K_{t+1})$  is a non-monotone function in  $K_{t+1}$ , we obtain the function

$$K_t = \psi^{-1}[\gamma(K_{t+1})] \equiv G(K_{t+1}).$$

The function  $G(\cdot)$  has the inverted U shape if  $\gamma(K_{t+1})$  does. In such cases, by a suitable choice of the function  $\psi$  (via the choice of the production function), we can ensure that the map  $G(K_{t+1})$  generates chaotic behavior for the backward dynamics. (See Appendix 1 for a precise statement on chaotic equilibria.) We summarize this result in the following proposition.

**Proposition 1:** Chaotic behavior is possible if the function  $G(\cdot)$  has the inverted U shape. For this to hold, it is necessary that the condition stated in lemma 5 be satisfied.

**Remark:** We can derive further properties of the  $G(\cdot)$  function in example 2. From (34)

$$\psi'(K_t) \frac{dK_t}{dK_{t+1}} = \left[ \frac{2 + \eta}{1 + \eta} \right] + K_{t+1} \frac{\eta'}{(1 + \eta)^2}$$

At  $(K_{t+1}, K_t) = (0, 0)$ , the slope  $dK_t/dK_{t+1} = 2/\psi'(0) = 0..$  Thus, the stationary point  $(K_{t+1}, K_t) = (0, 0)$  is unstable in backward dynamics, and stable in forward dynamics. Note that  $G'(\cdot)$  changes sign once if and only if there exists a unique value  $K_{t+1}^C > 0$  at which

$$\left[ 1 + \eta(K_{t+1}^C) \right] \left[ 2 + \eta(K_{t+1}^C) \right] = K_{t+1}^C \eta'(K_{t+1}^C) \quad (35)$$

## 5 Comparison with passive financial intermediation

Under “passive intermediation”, the FI that lends to entrepreneur  $\theta$  in period  $t$  does not know how much capital the entrepreneur  $\theta$  actually invests in the risky project. It does know, in period  $t + 1$ , whether the project is successful (i.e., the return from the project is positive), or not (the return is zero.) In the case of success, the entrepreneur pays back to the FI the amount  $R_{t+1}B_t(\theta)$ , and in the case of failure, he pays nothing. We assume that the FIs know the aggregate (i.e., economy-wide) level of investment in the risky projects,

which we denote by  $K_{t+1}^A$  (where the superscript  $A$  stands for “aggregate”). They can therefore deduce the average rate of non-default,  $p^A(K_{t+1}^A)$ .

Since  $K_{t+1}(\theta)$  is not observed by the FI, the gross rate of interest  $R_{t+1}$  that the FI charges entrepreneur  $\theta$  is independent of his action. Since the FIs break even in equilibrium, we have

$$p^A(K_{t+1}^A)R_{t+1} = r_{t+1} \quad (36)$$

where  $r_{t+1}$  is the rate the FIs pay to their depositors.

The entrepreneur  $\theta$  chooses  $S_t(\theta)$ ,  $K_{t+1}(\theta)$ ,  $B_t(\theta)$  to maximize his expected utility:

$$EU = U_1 [C_t^y(\theta)] + p(K_{t+1}(\theta)) V_2 [C_{t+1}^{oG}(\theta)] + [1 - p(K_{t+1}(\theta))] V_2 [C_{t+1}^{oB}(\theta)] \quad (37)$$

where

$$C_t^y(\theta) = W_t + B_t(\theta) - K_{t+1}(\theta) - S_{t+1}(\theta) \quad (38)$$

$$C_{t+1}^{oG}(\theta) = r_{t+1}S_t(\theta) + f'(\phi(W_{t+1}))K_{t+1}(\theta) - R_{t+1}B_t(\theta) \quad (39)$$

and

$$C_{t+1}^{oB}(\theta) = r_{t+1}S_t(\theta) \quad (40)$$

Notice that the variable  $R_{t+1}$  is not indexed with  $\theta$  because the FI's loan contract does not specify  $K_{t+1}(\theta)$ .

We now substitute (38), (39) and (40) into (37), and differentiate the resulting expression with respect to  $S_t(\theta)$ ,  $K_{t+1}(\theta)$ ,  $B_t(\theta)$ . We then obtain the following first order conditions. With respect to  $B_t(\theta)$  :

$$U'_1 = p(K_{t+1}(\theta)) V'_2 [C_{t+1}^{oG}(\theta)] R_{t+1} \quad (41)$$

with respect to  $S_t(\theta)$  :

$$\begin{aligned} U'_1 &= p(K_{t+1}(\theta)) V'_2 [C_{t+1}^{oG}(\theta)] r_{t+1} + \\ &[1 - p(K_{t+1}(\theta))] V'_2 [C_{t+1}^{oB}(\theta)] r_{t+1} \end{aligned} \quad (42)$$

with respect to  $K_{t+1}(\theta)$  :

$$\begin{aligned} U'_1 &= p(K_{t+1}(\theta)) V'_2 [C_{t+1}^{oG}(\theta)] f'(\phi(W_{t+1})) + \\ &p'(K_{t+1}(\theta)) [V_2 [C_{t+1}^{oG}(\theta)] - V_2 [C_{t+1}^{oB}(\theta)]] \end{aligned} \quad (43)$$

In a symmetric equilibrium, all entrepreneurs behave identically, hence

$$p(K_{t+1}(\theta)) = p^A(K_{t+1}^A)$$

This equation and (36) implies that (41) may be rewritten as

$$U'_1 = V'_2 [C_{t+1}^{oG}(\theta)] r_{t+1} \quad (44)$$

From (44) and (42), we get

$$V'_2 [C_{t+1}^{oG}(\theta)] = V'_2 [C_{t+1}^{oB}(\theta)] \quad (45)$$

hence

$$C_{t+1}^{oG}(\theta) = C_{t+1}^{oB}(\theta) = r_{t+1} S_t(\theta) \quad (46)$$

implying that old-age consumption is independent of the state of nature. That is, the consumer is perfectly insured. Thus risk-averse agents shift the entire investment risk to the risk-neutral FIs. Equation (46) implies that

$$f'(\phi(W_{t+1})) K_{t+1}(\theta) = R_{t+1} B_t(\theta) \quad (47)$$

Now, (44) and (43) imply that

$$r_{t+1} = p(K_{t+1}(\theta)) f'(\phi(W_{t+1})) \quad (48)$$

Thus, we obtain:

**Lemma 6:** Under passive financial intermediation, the safe rate of return  $r_{t+1}$  is equal to the expected rate of return in the risky projects.

**Remark:** This result is in sharp contrast with Lemma 3.

**Lemma 7:** Under passive financial intermediation, in equilibrium, the amount each entrepreneur invests in the risky project is equal to the amount he borrows, i.e.,  $K_{t+1} = B_t$ .

**Proof:** Multiply (47) by  $p$  to get

$$p R_{t+1} B_t(\theta) = p f'(\phi(W_{t+1})) K_{t+1}(\theta)$$

This and (36) give

$$r_{t+1} B_t(\theta) = p f'(\phi(W_{t+1})) K_{t+1}(\theta) \quad (49)$$

From (49) and (48)

$$B_t(\theta) = K_{t+1}(\theta). \quad (50)$$

**Remark:** This result is in sharp contrast with Lemma 4.

We now turn to the dynamics of the model. From (44) and (46), we have

$$U'_1 = r_{t+1} V'_2 [r_{t+1} S_t] \quad (51)$$

Now, in a symmetric equilibrium,  $S_t = B_t$ . Using this and (50), we get  $S_t = K_{t+1}$ . Substitute this result into (51) to obtain

$$U'_1 [W_t - K_{t+1}] = p(K_{t+1}) f'(\phi(W_{t+1})) V'_2 [p(K_{t+1}) f'(\phi(W_{t+1})) K_{t+1}] \quad (52)$$

Since (6) remains true, (23) is still valid in this section. Thus (52) may be written as

$$U'_1 [\psi(K_t) - K_{t+1}] = p(K_t) f'(p(K_{t+1}) K_{t+1}) V'_2 [p(K_{t+1}) f'(p(K_{t+1}) K_{t+1}) K_{t+1}] \quad (53)$$

This first order difference equation gives us the dynamics of the model. In the case of log utility, it reduces to

$$\frac{1}{\psi(K_t) - K_{t+1}} = \frac{1}{K_{t+1}}$$

This yields  $K_{t+1} = H(K_t)$  where  $H$  is monotone increasing. Therefore *fluctuations are impossible*. We can now state:

**Proposition 2:** With the log utility function, it is impossible to have chaotic dynamic behavior in the passive financial intermediation regime.

## 6 Conclusion

This paper focuses on the possibility of endogenous fluctuations caused by activities of financial intermediaries within the context of a simple overlapping generations model. Risk-averse agents face idiosyncratic income losses, the probability of which they can affect through their own capital investments. We showed that in the economy with active intermediaries, the optimal loan contract achieves complete risk sharing but, given the loan size, the amount invested in the risky project is lower in the presence of active intermediaries than otherwise. This last fact alone creates an income effect which is responsible for the endogenous generation of cycles and chaotic dynamics. The analysis indicated that in the absence of active financial intermediation, the economy studied would not exhibit any complex dynamics.

It bears emphasis that all these results are derived within the context of a complete information environment and without recourse to preference-shocks a la Diamond and Dybvig (1983). In fact, were we to incorporate unobservable liquidity shocks to preferences in our model, our results would actually be strengthened. These shocks would then constitute another source of distortions; consequently, old-age consumption would no longer be non-stochastic in such a framework. Since our emphasis is on the role of active intermediation in the creation of chaos, there is no point in compounding sources of complex dynamics.

Casual empiricism would suggest that richer countries (arguably these are countries with the most developed financial systems) are subject to more (possibly more violent) cyclical fluctuations than poorer countries. If one makes the speculative connection that the active intermediation regime discussed above reflects those that are present in the richer countries, then our analysis, several caveats notwithstanding, may be interpreted as providing a partial explanation for this observation.

A caveat needs to be recorded. The set-up studied above assumed that capital investments by agents were perfectly observable to any other agent. Also, the riskiness of each project was the same. In other words, problems of moral hazard and adverse selection were assumed away. Doubtless these omissions are glaring departures from reality. However, to make a theoretical point, we find it neater to avoid unnecessary complexity.

## Appendix 1: Chaotic Equilibria

Let  $X$  be a closed and convex subset of the real number line. Let  $G$  be a continuous function that maps  $X$  into  $X$ . The pair  $(X, G)$  is called a dynamic system. Let  $G^n$  denote the  $n$ th iterate of  $G$ . A point  $k^*$  is called a periodic point of  $(X, G)$  with period  $m$  (where  $m$  is an integer), if  $G^m(k^*) = k^*$  but  $G^n(k^*) \neq k^*$  for  $n = 1, 2, \dots, m - 1$ . The point  $k^*$  is then said to generate period- $m$  cycles.

A subset  $Y$  of  $X$  is called a *scrambled set* of the dynamic system  $(X, G)$  if  $Y$  has the following properties:

- (a)  $Y$  has an uncountable number of points,
- (b)  $Y$  does not contain any periodic points of the dynamic system  $(X, G)$ ,
- (c) for any  $u, v \in Y$ , where  $u \neq v$ ,

$$\limsup_{i \rightarrow \infty} \|G^i(u) - G^i(v)\| > 0 \text{ and } \liminf_{i \rightarrow \infty} \|G^i(u) - G^i(v)\| = 0,$$

- (d) for any periodic point  $k^* \in X$  and any  $u \in Y$

$$\limsup_{i \rightarrow \infty} \|G^i(u) - G^i(k^*)\| > 0.$$

If a dynamic system  $(X, G)$  has a scrambled set, we say that the dynamic system is *chaotic*.

Property (d) means that the orbit generated by any point in the scrambled set does not converge to a limit cycle. Property (c) means that any two points from the scrambled set will generate two paths that will eventually get very close together in one sense, yet remain far apart, in another sense.

The following theorem by Li and York (1975) states the connection between chaos and the existence of period 3 cycles:

**Theorem:** If there is a point  $x \in X$  such that

$$G^3(x) \leq x < G(x) < G^2(x) \text{ or } G^3(x) \geq x > G(x) > G^2(x) \quad (54)$$

- then (i) for every positive integer  $m$ , there is a periodic point of period  $m$ ,
- (ii) there is a scrambled set  $Y$  in  $X$ .

Clearly, we can apply this theorem to our model. The set  $X$  of feasible capital stock levels can be made bounded by stipulating that  $p(K) = 0$  for

$K \geq \bar{K}$  ( a certain positive stock level). The inequalities (54) given in the theorem can be met by appropriate restrictions on the  $p(\cdot)$  function.

### **Appendix 2: Example of chaotic dynamic system**

By suitable specification of the  $p(\cdot)$  function, it is possible to generate a logistic map. The logistic map  $k_{t+1} = \mu k_t(1 - k_t)$ , where  $0 < \mu \leq 4$  and  $k \in [0, 1]$ , can generate chaotic dynamic by an appropriate choice of  $\mu$  (Devaney, 1986, Rasband, 1990). This equation has a unique positive stationary solution  $\bar{k}$ . If  $\mu \in (4 - 2\sqrt{2}, 3)$  then all trajectories starting from a positive  $k_0$  converge to  $\bar{k}$ . This convergence exhibits damped oscillations if  $2 < \mu < 3$ . At  $\mu = 3$  the steady state loses its stability and a stable period 2 cycle is born. This is the beginning of the period-doubling route to chaos, which culminates at  $\mu \approx 3.57$ . Beyond this value, there exist parameter regions with completely chaotic behavior. The set of parameter values for which the map generates chaotic trajectories has positive Lebesgue measure.

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