Estimation of Bilateral Exposures - A Copula Approach

An Extended Abstract and Brief Overview

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The recent upswing in the popularity of economic applications of 'networks' may be attributed to the relative efficiency of network theory in analyzing complex economic structures with much more clarity compared to any other (standard) methods used by economists. Having said that, it can hardly be denied that most of the network-based economic models suffer from lack of availability of 'ready-to-use' network datasets. As such, a number of times, researchers are forced to base their analyses on simulations or pick from the small number of datasets available (for example, ICPSR - Add Health Dataset). This tends to limit the variety of empirical exercises that could be carried out to understand some economic networks better. For instance, consider an interbank network. Most of the datasets with public access do not have information about the bilateral connections between banks within this network. Under such circumstances, conducting any form of predictive or explanatory analyses with regard to the manner in which connections are formed in such a network is almost impossible. Upper and Worms (2004) addressed this issue by using the technique of Maximum Entropy (ME) to estimate these connections from the (publicly available) information on total assets and liabilities of the banks in the German interbank network. However, as will be discussed shortly, ME has been proven to have some obvious shortcomings. This serves as the motivation for our current research.

In this paper, we construct a 'copula' based approach instead, and establish its merits over the Maximum Entropy approach. More than an all purpose methodology, our paper aims to extend the toolkit of financial stability evaluation that allows an analyst to make different

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assumptions with respect to the dependence of the exposures desirably rooted in a well founded theoretical framework. Such an instrument promises to be a valuable contribution to stress testing exercises based on a familiar tool of risk management such as copulas.

First, we provide a brief overview of the data that we use. The recent literature on contagion using bilateral exposures (e.g., Blavarg and Nimander 2001, Furfine (2003), Wells (2004), Cocco et al. 2009) was predominantly written by supervisors given the data availability. The notable exception comes from studies based on the e-MID dataset. e-MID is the Italian based European reference electronic market for liquidity trading. This platform provides 'anonymized' data for euro denominated unsecured interbank transactions. Although it, recently, experienced a strong decline, e-MID accounted for 17% of overall Euro area unsecured money market turnover in 2006 (see, ECB 2007). Several papers have been written using this unique dataset. A selected list of papers and contributions follows. Iori et al. (2007) use network analysis to identify communities in liquidity trading activity, and later, Iori et al. (2008) map the network topology of this segment. Besides, this dataset also opens up the possibility of exploring the strategic behavior of market participants. Cohen-Cole et al. (2011) develop a model with strategic elements that capture shock propagation within the interbank network. Moreover, Gabrieli (2011) sheds light on how banks perceive the position of their neighbors in the interbank network and tests whether nodes' connections reflect the too-interconnected-to-fail implicit government guarantee. This brief literature review shows that network theory can provide a solid support to analyze increasingly important issues. However in order to do so, micro-data is required underlining the need for methods that are able to provide estimates of bilateral exposures given the available data. We will use the e-MID dataset to establish a methodology that estimates the bilateral exposures in addition to the method of Maximum Entropy. Further, in order to establish that our method works, we use the Bank of International Settlements (BIS) dataset that provides us with information on inter-country bilateral as well as total exposures. Once, we test our method on this data, we can then apply it to the e-MID data to estimate the corresponding bilateral exposures.

The methodology that we propose is based on *copulas*. A copula is a cumulative distribution function with uniform marginals. In cases where we may only have access to the marginal distributions, a copula can express the dependence structures using the transformed marginals as a reference case. In particular, a copula-based approach, can serve to be very insightful when we have information about the total liabilities and total assets of the banks in an interbank market and would like to estimate the bilateral connections. The underlying algorithm works as follows. For example, suppose we only had information about the marginals. In order to extract out the bilateral connections from this, we need to fit a copula to the data. For that, we first transform the data into uniform distribution, which is required for it to be able to serve as an input into the copula function. Thereafter, we fit a copula to the transformed data using maximum likelihood estimation. The choice of copula varies according to the priors that a researcher might have about the data and the kind of restrictions he/she would like to impose. Since, a copula is a cumulative distribution function, it generates the probabilities of these bilateral connections, which can then be used to simulate stochastic matrices or adjacency matrices by imposition of a cut-off rule. This, brief description of the methodology, provides a flavor of the advantages and flexibilities that copulas have over maximum entropy. In particular, the malleability of copulas with respect to imposition of dependency structure (hierarchical, mixed, etc.) is extremely attractive and enhances the possibility of an estimation that fits well. We intend to exploit such advantages of copulas to construct a much more effective method of estimating bilateral exposure as compared to Maximum entropy as proposed by Upper and Worms (2004). Further, the standard maximum entropy estimation consists in dividing equally among all other nodes the total exposures and then use the RAS algorithm (Schneider and Zenios 1990) to re-balance the matrix. Matrix re-balancing is required to ensure that the sum of the individual elements amount to the original overall exposures. The copula based procedure also uses the RAS algorithm but differs in the manner it treats the input matrix. Instead of just treating all individual exposures equally, our proposed method uses the values assigned by the copula to generate a stochastic matrix. Then, this matrix is multiplied by total exposures to obtain an estimate of the bilateral values. This method ensures that the restrictions on the connections are imposed such that we get a good fit.

In order to assess the performance of the proposed estimator a goodness-of-fit measure is required. Therefore, we adopt a distance-based error measure that aggregates the inconsistencies between the estimated adjacency (or exposure) matrix and the observed one (from the BIS dataset). Consider an arbitrary exposure between two nodes estimated using the procedure described before $\hat{a}_{ij}$, then the error measure is nothing but its distance to the the actual one a_{ij}. Aggregating across all nodes and normalizing by the total exposure, we get:

$$\epsilon = \frac{\sum_i \sum_j |\hat{a}_{ij} - a_{ij}|}{\sum_i \sum_j a_{ij}}. \tag{1}$$

The underlying idea is that whether the estimator produces a guessed value that lies above or below the actual one, an error is incurred. The larger is the error measure the larger will be the inconsistencies between the estimated and the observed matrix. Even though in the current form this measure weights each inconsistency identically, a weighted or quadratic measure can be obtained straightforwardly. It is also worth noting that since the measure is normalized, when comparing the results obtained with different estimated methods the improvements obtained by choosing one method instead of another always relate to the overall exposure of the system.

Since this is research is currently in progress, we have chosen to report the results from a preliminary set of simulations that we conducted using a 'Gumbel' copula in order to determine conditions under which the procedure based on the Gumbel copula outperforms the 'traditional' maximum entropy estimation. The theoretical apparatus that served as a base for this exercise was the core-periphery model. Boss et al. (2004) found that the degree distribution of the Austrian interbank network follows power laws, i.e., a few nodes maintain a large number of connections while a large number of nodes have only few connections. Thus, we can theorize an exposure matrix where few nodes hold a substantial amount of cross exposures (core) while maintaining much weaker connections with the remaining ones (periphery). In fact, Craig and von Peter (2010) provides some evidence of this structure on the German interbank market. This certainly does not seem to be an innocuous assumption. Therefore, in an early stage of a theoretical network formation model, it is essential to ensure that the estimated exposure matrix is meaningful.

This abstract model has four blocks: core-core CC, core-periphery CP, periphery-core PC and periphery-periphery PP relations. To ilustrate this framework, we can look at Bank of International Settlements data. Here, the top and bottom 5 countries in terms of overall exposures are the following:

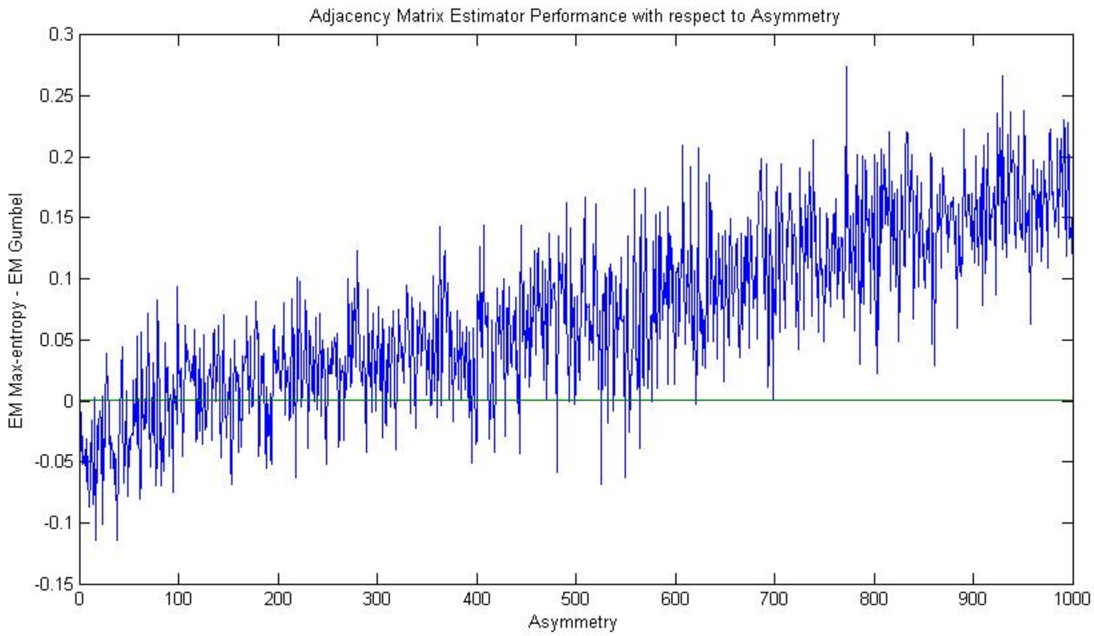
Table 1: Top and Bottom 5 Countries Cross Exposures

Country	US	GB	DE	FR	IT	FI	TR	SE	GR	CL
US	**0**	**1095468**	**491136**	**529732**	**35143**	*379*	*4781*	*35410*	*5434*	*1558*
GB	**520953**	**0**	**465682**	**281450**	**46050**	*2324*	*2790*	*43516*	*13441*	*64*
DE	**175515**	**175165**	**0**	**260927**	**257408**	*2557*	*4152*	*77171*	*5185*	*55*
FR	**202470**	**269083**	**189401**	**0**	**42440**	*3434*	*1578*	*12794*	*2089*	*58*
IT	**35071**	**66387**	**162285**	**392577**	**0**	*711*	*439*	*1094*	*584*	*24*
FI	10400	3943	13773	7860	1198	***0***	***1***	***121991***	***2***	***2***
TR	19745	24682	18306	23873	4017	***0***	***0***	***234***	***30451***	***5***
SE	22778	19033	34244	13119	2745	***3405***	***166***	***0***	***24***	***8***
GR	7320	14060	26059	56740	4085	***27***	***107***	***145***	***0***	***0***
CL	7846	2948	4542	3318	813	***0***	***0***	***100***	***0***	***0***

Source: BIS - ultimate risk basis dataset

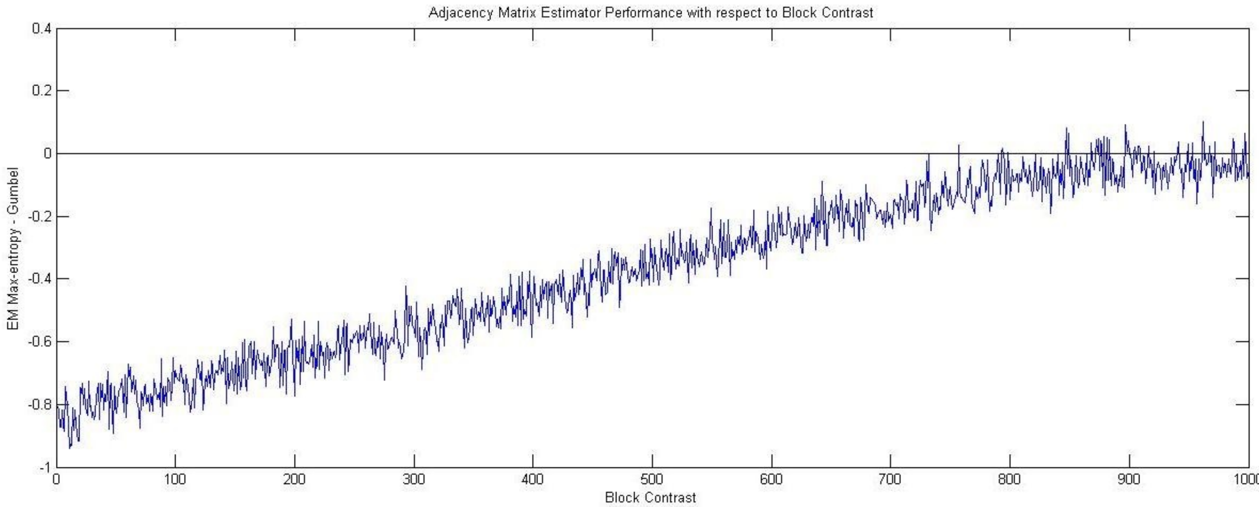
It is easy to see that the cross-exposures established between the top countries are more substantial than the ones maintained in the remaining blocks. Moreover, the matrix is asymmetric, i.e., peripheral countries tend to lend more to the core than the other way around (this fact has also been observed by (Cocco et al. 2009) for the Portuguese interbank market). Finally, the block or sub-matrix that describes the relations between peripheral countries is quite sparse. Thus, the simulations focus on two features: asymmetry and contrast between blocks. To generate an asymmetric matrix, with the assumed block structure, a random number generator was used. The idea is to simulate higher exposures in the CC block and then impose a asymmetric structure by adding positive random numbers in all entries below the diagonal. Thus, the greater is the magnitude of this additional perturbations the more asymmetric the matrix is. As a result, the difference between the error measures obtained following the maximum entropy procedure vis-‘a-vis our proposed gumbel copula driven methodology increases as the original matrix becomes more asymmetric. Figure 1 displays the results of the described simulation. As we can see the series of the differences of the error measures of the two methods cross the horizontal line of equality as asymmetry grows. Note that this difference is measured on an overall exposure basis, i.e., when asymmetry is very large the error measured obtained with the copula based procedure produces a smaller sum of inconsistencies that amount to about 15-25% of the total exposure.

Figure 1: Asymmetry



Similarly, we can generate a matrix where the CC relations are much stronger than the ones lying outside of this block. In order to do so, it suffices to generate a set of random number such that the contrast between the CC block and the remaining ones is increasing. Figure 2 shows that as the contrast increases a Gumbel copula estimation improves comparatively to a maximum entropy one. Thus, the comparatively stronger the relations established by core nodes the better our proposed estimator performs.

Figure 2: Contrast Between Blocks



Given that BIS data is the only publicly available dataset where bilateral exposures can be observed, we test our method using the matrix that exemplifies the core-periphery structure. We find that the copula approach produces a slightly lower error measure (1.1662 vs 1.1849) than the one obtained with the standard maximum entropy approach. Thus, we obtain a precision 'gain' that amounts to roughly 2% of total exposures. Tables 2 and 3 display the estimates. Given that the Gumbel copula is asymmetric and heavy tailed, it is not surprising to find that the copula approach produces closer estimates (in bold) in the CC and CP block while performing worst in the PP block.

Table 2: Exposure Matrix Estimated Using a Gumbel Copula Based Approach

Country	US	GB	DE	FR	IT	FI	TR	SE	GR	CL
US	0	**779720,4**	**558819,6**	611760,2	**133285,8**	**3865,5**	4190,0	**89741,3**	**17140,1**	518,1
GB	350526,3	0	**405226,9**	434518,2	**99580,6**	**2899,2**	**3142,6**	**67136,0**	12851,6	**388,6**
DE	231484,2	**331229,5**	0	**267727,6**	**68123,7**	**2015,7**	**2184,9**	**46175,3**	8923,9	270,3
FR	**180278,4**	245604,8	**192644,8**	0	55578,7	**1692,3**	1834,2	38013,0	7473,8	227,0
IT	137937,9	**185739,1**	**146138,0**	150859,4	0	1332,1	1443,8	29666,1	5876,8	178,8
FI	31789,6	**40200,8**	**32086,0**	**32766,6**	11536,9	0	480,8	**8342,4**	1907,1	60,0
TR	24227,1	**30550,9**	**24398,2**	**24904,7**	8879,8	351,0	0	6452,3	**1501,6**	47,4
SE	**20085,1**	**25283,0**	20198,5	**20612,2**	7408,9	**298,3**	323,0	0	1272,6	40,4
GR	21878,4	**27564,6**	22017,3	22471,3	8044,7	320,9	347,5	5854,9	0	43,4
CL	**3891,1**	**4875,8**	3898,9	**3975,9**	1459,9	62,1	67,2	1073,7	262,4	0
Source: Authors' calculations										

Table 3: Exposure Matrix Estimated Using Maximum Entropy

Country	US	GB	DE	FR	IT	FI	TR	SE	GR	CL
US	0	748778,6	565554,4	606587,6	147321,4	4470,5	4856,8	101068,3	19796,5	606,9
GB	362811,9	0	395215,2	423889,6	102949,7	3124,0	3394,0	70627,5	13834,0	424,1
DE	231084,7	333274,7	0	269986,7	65571,4	1989,8	2161,7	44984,6	8811,2	270,1
FR	177848,2	256496,0	193732,0	0	50465,3	1531,4	1663,7	34621,2	6781,3	207,9
IT	133117,5	191984,5	145006,4	155527,2	0	1146,2	1245,3	25913,6	5075,7	155,6
FI	30451,8	43918,1	33171,5	35578,2	8640,8	0	284,9	5928,0	1161,1	35,6
TR	23212,4	33477,4	25285,6	27120,1	6586,6	199,9	0	4518,7	885,1	27,1
SE	18949,4	27329,2	20641,8	22139,5	5377,0	163,2	177,3	0	722,5	22,2
GR	20884,0	30119,2	22749,1	24399,7	5925,9	179,8	195,4	4065,4	0	24,4
CL	3738,2	5391,2	4072,0	4367,5	1060,7	32,2	35,0	727,7	142,5	0
Source: Authors' calculations										

To the best of our knowledge, the methodology proposed to estimate bilateral exposures in our paper is completely new and innovative. Since this is an ongoing research we are likely to construct a more advanced model that could be either a 'mixture of copulas' or 'hierarchical copulas' in order to be able to tap the dependence structure from the available marginals. This is driven by the fact that there are blocks within the network that are more likely to be inter-connected compared to other blocks (such as core-core, as mentioned above) and so on. A mixture or hierarchical model of copulas can prove to be a better fit when such complexities underscore the data.

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